APPLICATION OF BSS TO SPACE-TIME CODED WIRELESS COMMUNICATIONS SYSTEMS

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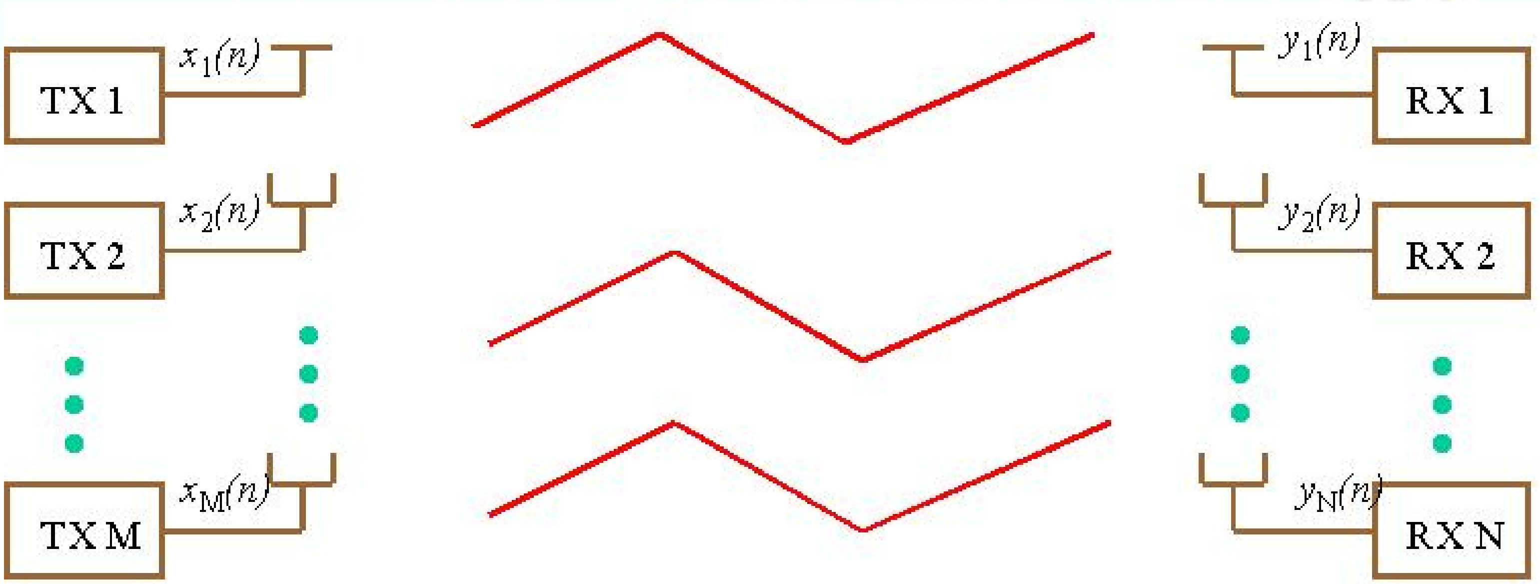
ICA Discussion Meeting, Queen Mary University of London, Dec. 2002

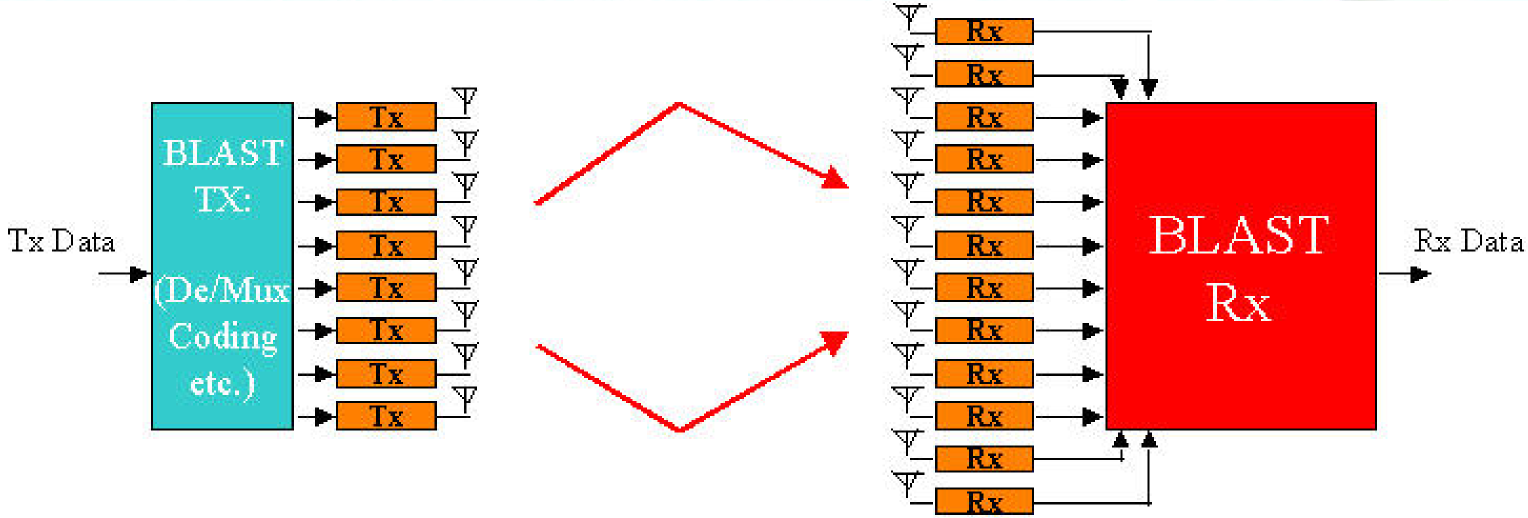
Multiple Antennas for Wireless Communications



- Use multiple antennas at both transmitter and receiver increases capacity

BLAST Transceiver Architecture

G. Foschini, Bell Labs Tech. J., 1996



- BLAST – Bell Labs Layered Space Time
- Different data sub-streams are transmitted from different antennas
- Signal processing at receiver attempts to separate the received signals

V-BLAST Receiver Processing

- Signal model (M transmit, N receive antennas, $N \geq M$):

$$\mathbf{y}(k) = \mathbf{H}\,\mathbf{s}(k) + \mathbf{n}(k)$$

where

$\mathbf{y}$ is $N \times 1$ vector of received signals
$\mathbf{s} = [s_1, \ldots, s_M]^T$ is $M \times 1$ vector of transmitted signals
$\mathbf{H}$ is $N \times M$ channel matrix
$\mathbf{n}$ is $N \times 1$ additive noise vector

- Linear receivers:

$$\mathbf{z}(k) = \mathbf{W}^T\,\mathbf{y}(k)$$

  – Decorrelating (ZF) receiver: $\mathbf{W} = \mathbf{H}^*[\mathbf{H}^T\mathbf{H}^*]^{-1}$
  – MMSE receiver: $\mathbf{W} = [\mathbf{H}^*\mathbf{H}^T + M/\rho\mathbf{I}]^{-1}\mathbf{H}^*$

- Non-linear receivers: e.g., successive interference cancellation

Application of BSS to Receiver Design

- Received signals (in absence of noise):

$$\mathbf{y} = \mathbf{H}\mathbf{s}$$

- Separated outputs:

$$\mathbf{z} = \mathbf{W}^T\mathbf{y} = \mathbf{W}^T\mathbf{H}\,\mathbf{s} = \mathbf{G}^T\mathbf{s}$$

- Assumptions:
  1. Each $s_i(k), i = 1, \ldots, M$ is non-Gaussian iid with zero-mean
  2. $\{s_i(k)\}$ and $\{s_j(k)\}$ are statistically independent for $i \neq j$ and share same statistical properties
  3. $\mathbf{H}$ has full column rank

- Blind recovery is achieved if (after suitable reordering):

$$z_i(k) = e^{j\phi_i} s_i(k), \quad i = 1, \ldots, M, \quad \phi_i \in [0, 2\pi)$$

- Structure of $\mathbf{G}$ for blind recovery:

$$\mathbf{G}_i = \left[0 \cdots 0 \;\; e^{j\phi_i} \;\; 0 \cdots 0\right]^T$$

Kurtosis Maximization

C. Papadias, IEEE Trans. SP, 2000

- Cost function:

$$J_{\text{MUK}} = \sum_{i=1}^{M} |K(z_i)|$$

subject to $E(\mathbf{z}\mathbf{z}^H) = \sigma_s^2 \mathbf{I}$

where $K(z) = E(|z|^4) - 2E^2(|z|^2) - |E(z^2)|^2$ is (unnormalized) kurtosis

- Recall:

$$\mathbf{z} = \mathbf{W}^T \mathbf{y} = \mathbf{W}^T \mathbf{H} \mathbf{s}$$

- For unitary channel matrix:

$E(\mathbf{z}\mathbf{z}^H) = \sigma_s^2 \mathbf{I}$ is satisfied by $\mathbf{W}^H \mathbf{W} = \mathbf{I}$

- Algorithm:
  1. Stochastic gradient update:

$$\mathbf{W}'(k+1) = \mathbf{W}(k) + \mu \, \nabla_{\mathbf{W}} J_{\mathrm{MUK}}(k)$$

  2. Orthogonalization :

$$\mathbf{w}_1(k+1) = \frac{\mathbf{w}_1'(k+1)}{\|\mathbf{w}_1'(k+1)\|}$$

$$\mathbf{w}_i(k+1) = \frac{\mathbf{w}_i'(k+1) - \sum_{l=1}^{i-1} (\mathbf{w}_l^H(k+1)\mathbf{w}_i'(k+1))\mathbf{w}_l(k+1)}{\|\mathbf{w}_i'(k+1) - \sum_{l=1}^{i-1} (\mathbf{w}_l^H(k+1)\mathbf{w}_i'(k+1))\mathbf{w}_l(k+1)\|}, \quad i = 2, \ldots, M$$

- Globally convergent

MIMO Constant Modulus Algorithm

L. Castedo, et al., IEEE Trans. SP, 1997

- Cost function:

$$J_{\text{CMA}} = E\sum_{i=1}^{M}(|z_i|^2 - R_p)^2 + \alpha \sum_{\substack{i,j=1 \\ i\neq j}}^{M} |E(z_i z_j^*)|^2$$

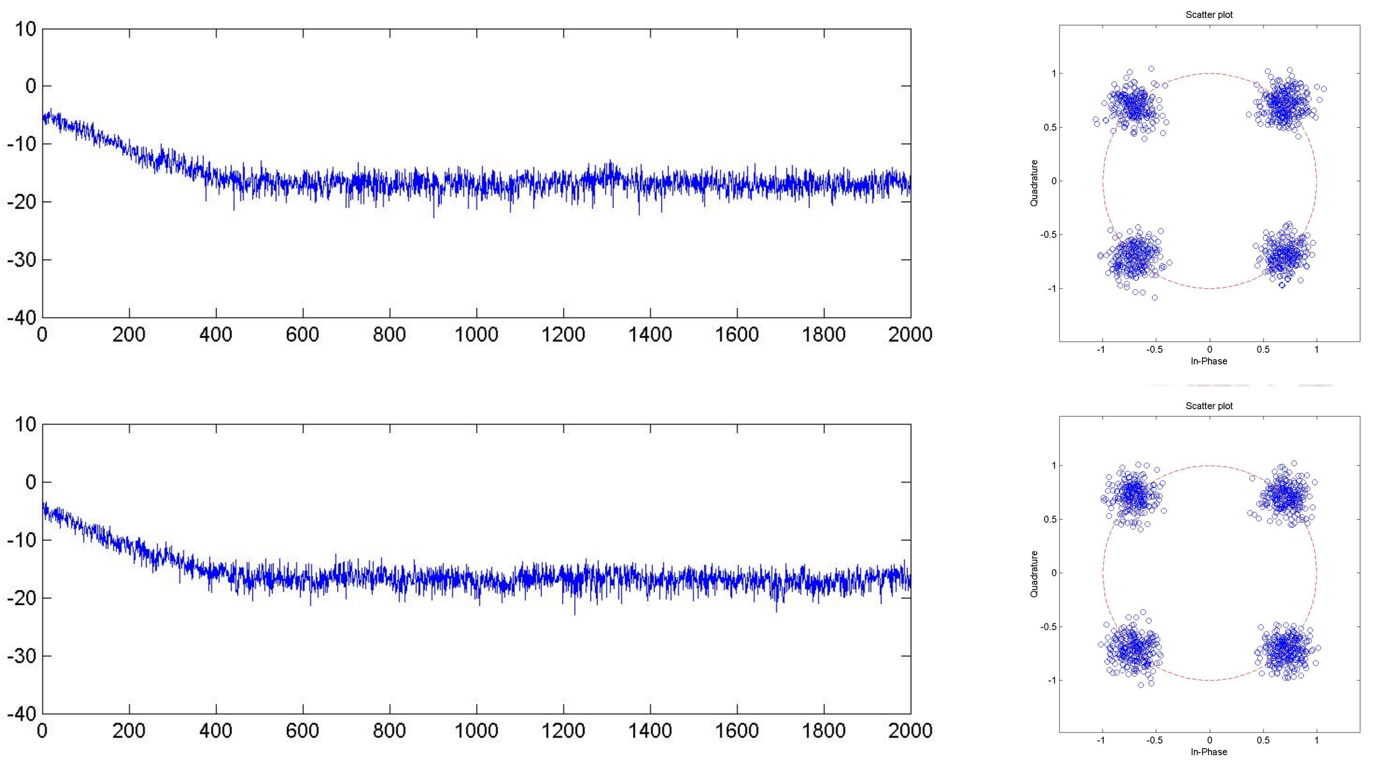
where $R_p = \frac{E|s(k)|^{2p}}{E|s(k)|^p}$

- Algorithm:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) - \mu\, \nabla_{\mathbf{w}_i} J_{\text{CMA}}(k)$$
$$z_i(k) = \mathbf{w}_i^T(k)\, \mathbf{y}(k)$$

- Globally convergent

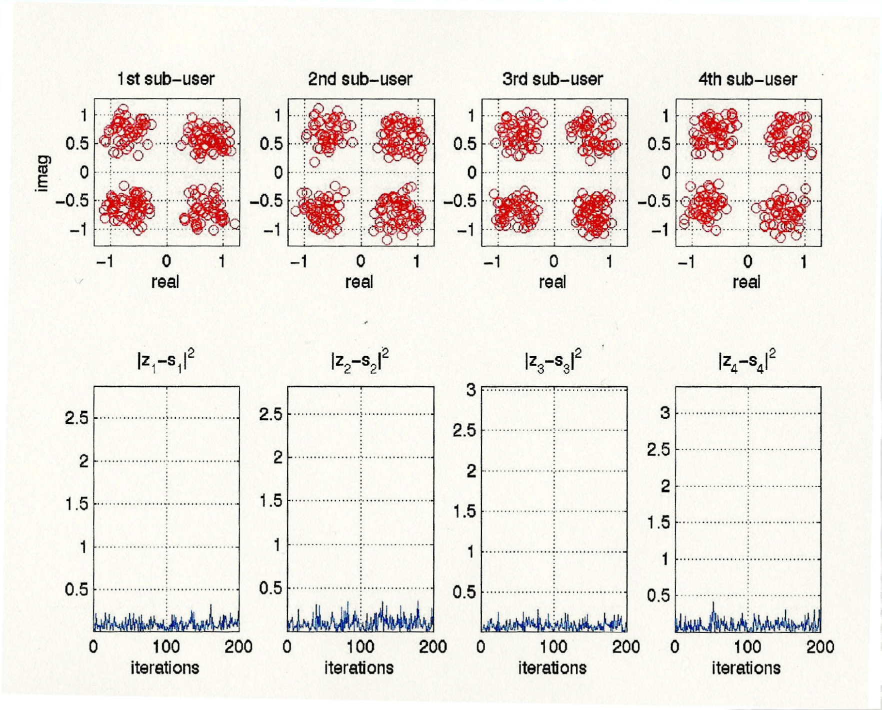
MUK Simulation Results



- $M = 2, N = 2$, SNR = 20 dB, $\mu = 0.01$

MUK Experimental Results

D. Samardzija et. al, GLOBECOM, 2001



(Results courtesy of Lucent Technologies)

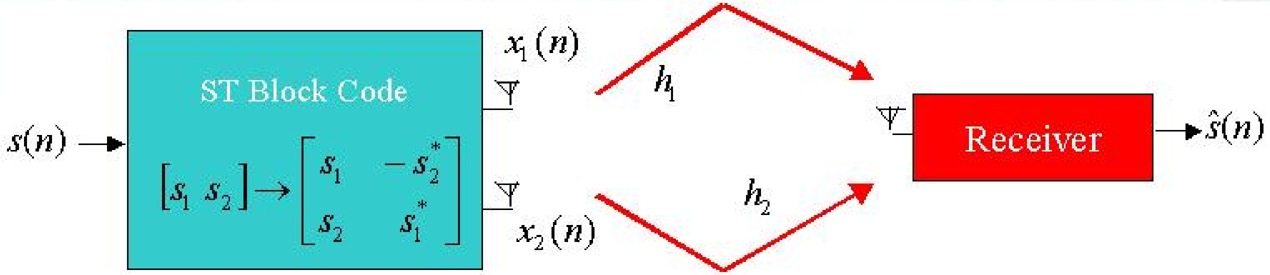
- After the 4th re-run: $M = 4, N = 6$, SNR ~ 12 dB, $\mu = 0.07$

Multiple Antennas

- Handsets:  Keep shrinking in size, multiple antennas (even 2) seem to be problematic
- PDAs:  Their size remains fixed (or even increases), 2 antennas may be reasonable
- Laptops:  Multiple antennas combined with BLAST-type processing may offer high data rates

Space-Time Block Coding

S. Alamouti, IEEE JSAC, 1998



- Input symbols grouped in twos, and transmitted from two antennas over two consecutive symbol periods
- Assuming channel is constant over consecutive symbol periods:

$$\begin{bmatrix} y(2k) & y(2k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} h_1 & h_2 \end{bmatrix}}_{\mathbf{h}} \begin{bmatrix} s_1(k) & -s_2^*(k) \\ s_2(k) & s_1^*(k) \end{bmatrix} + \begin{bmatrix} n(2k) & n(2k+1) \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} y(2k) \\ y^*(2k+1) \end{bmatrix} &= \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1(k) \\ s_2(k) \end{bmatrix} + \begin{bmatrix} n(2k) \\ n^*(2k+1) \end{bmatrix} \\ \mathbf{y}(k) &= \mathbf{H}\mathbf{s}(k) + \mathbf{n}(k) \end{aligned}$$

STBC Receiver

- Linear receiver:

$$\mathbf{z}(k) = \mathbf{W}^T \mathbf{y}(k)$$

- Channel matrix is orthogonal, $\mathbf{H}^H\mathbf{H} = \gamma I$, where $\gamma = |h_1|^2 + |h_2|^2$
- Let $\mathbf{W} = \mathbf{H}^*$:

$$\mathbf{z}(k) = \mathbf{W}^T\mathbf{y}(k) = \mathbf{H}^H\mathbf{H}\mathbf{s}(k) + \mathbf{H}^H\mathbf{n}(k) = \gamma\mathbf{s}(k) + \tilde{\mathbf{n}}(k)$$

$$\begin{aligned} z(2k) &= \gamma s_1(k) + [h_1^* n(2k) + h_2 n^*(2k+1)] \\ z^*(2k+1) &= \gamma s_2(k) + [h_2^* n(2k) - h_1 n^*(2k+1)] \end{aligned}$$

- Decoding rule:

$$\begin{aligned} \hat{s}_1(k) &= \arg\min_{s\in\mathcal{S}} |z(2k) - \gamma s|^2 \\ \hat{s}_2(k) &= \arg\min_{s\in\mathcal{S}} |z^*(2k+1) - \gamma s|^2 \end{aligned}$$

Application of BSS to STBC Receiver Design

- Receiver:

$$\mathbf{z} = \mathbf{W}^T \mathbf{y} = \mathbf{W}^T \mathbf{H}\,\mathbf{s} = \mathbf{G}^T \mathbf{s}$$

- Kurtosis maximization criterion:

$$\max_{\mathbf{W}} \sum_{i=1}^{M} |K(z_i)| \quad \text{subject to } \mathbf{G}^H \mathbf{G} = \mathbf{I}$$

- Because of structure of STBC:

$$\mathbf{G}^H \mathbf{G} = \mathbf{W}^H \mathbf{H}^* \mathbf{H}^T \mathbf{W} = \gamma \mathbf{W}^H \mathbf{W}$$

- Constraint becomes:

$$\mathbf{W}^H \mathbf{W} = \frac{1}{\gamma}\,\mathbf{I}$$

- Let

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \begin{bmatrix} \mathbf{g}_1 & \mathbf{g}_2 \end{bmatrix}$$

- We have

$$\begin{aligned} \mathbf{g}_2 &= \mathbf{J}\,\mathbf{g}_1^* \\ \mathbf{g}_1 &= \mathbf{J}^T\,\mathbf{g}_2^* \end{aligned}$$

  where

$$\mathbf{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- Impose this structure on the receiver

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 \end{bmatrix}, \quad \mathbf{w}_2 = \mathbf{J}\mathbf{w}_1^*$$

- So that

$$\mathbf{W}^H\mathbf{W} = \alpha\mathbf{I}, \quad \alpha = \|\mathbf{w}_1\|^2$$

- Normalize $\mathbf{w}_1$ such that $\alpha = 1/\gamma$

Kurtosis Maximization

- Algorithm:

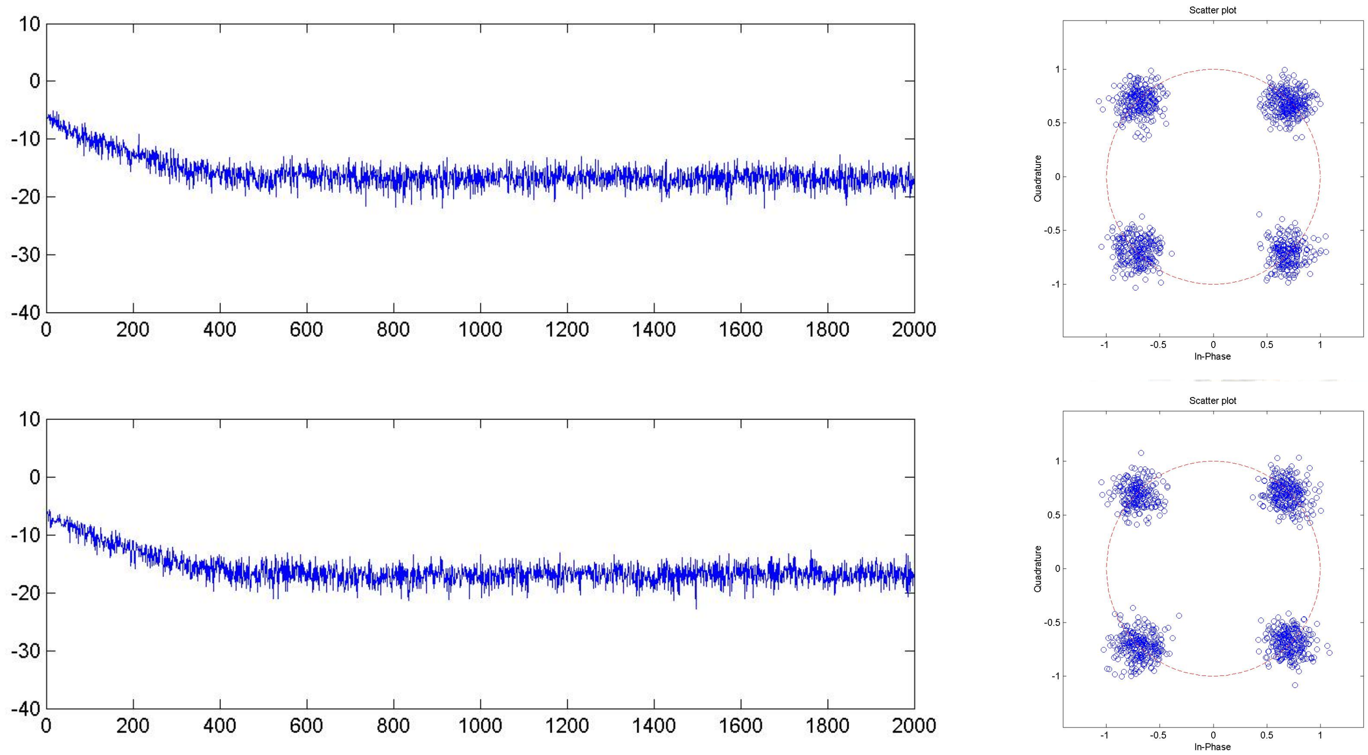
$$\begin{aligned}
z_1(k) &= \mathbf{w}_1^T(k)\mathbf{y}(k) \\
z_2(k) &= \mathbf{w}_2^T(k)\mathbf{y}(k) \\
\mathbf{w}_1^{'}(k+1) &= \mathbf{w}_1(k) + \mu\, \nabla_{\mathbf{w}_1} J_{\text{MUK}}(k) \\
\mathbf{w}_1(k+1) &= \frac{1}{\sqrt{\hat{\gamma}}} \frac{\mathbf{w}_1^{'}(k+1)}{\|\mathbf{w}_1^{'}(k+1)\|} \\
\mathbf{w}_2(k+1) &= \mathbf{J}\, \mathbf{w}_1^{*}(k+1)
\end{aligned}$$

- No pre-whitening required, γ is easily estimated as

$$\hat{\gamma}(k+1) = (1-\beta)\hat{\gamma}(k) + \beta\, \frac{\mathbf{y}^H(k)\mathbf{y}(k)}{\sigma_s^2}$$

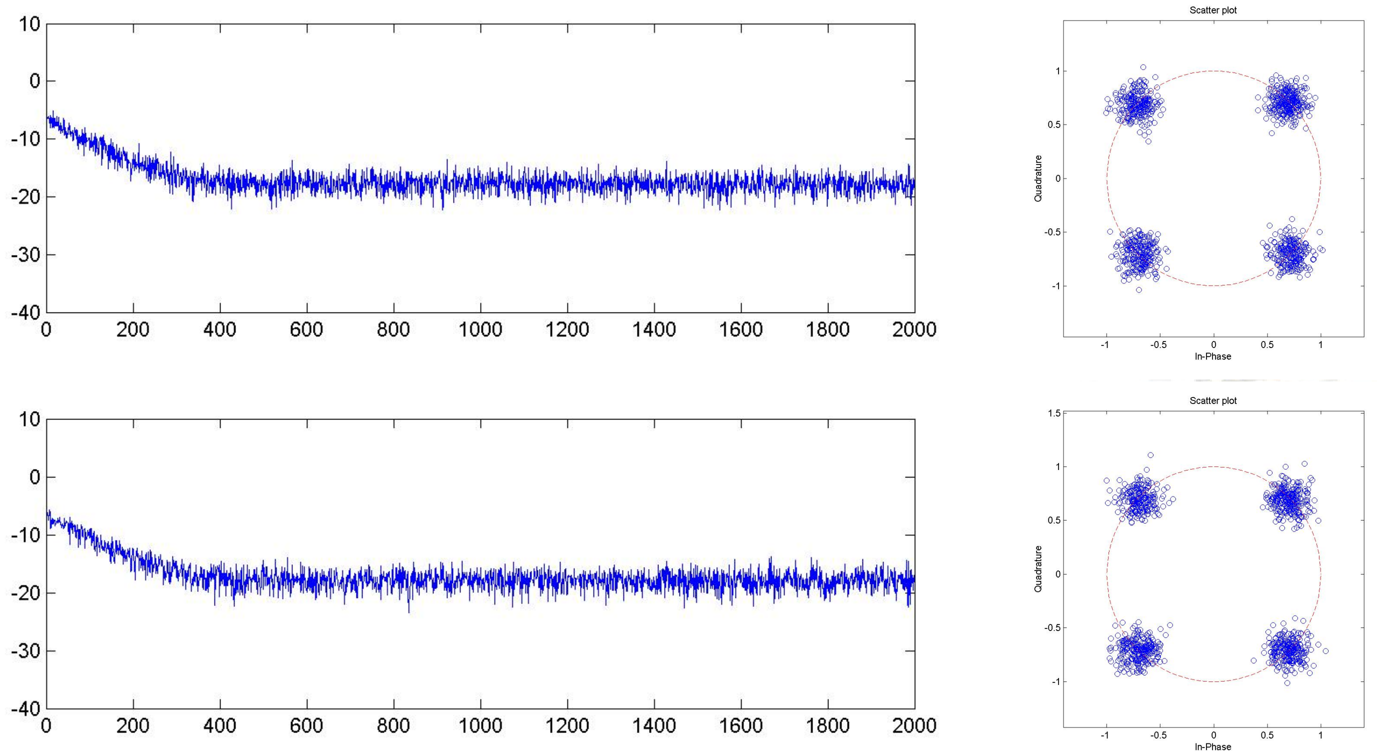
where $\beta \in [0,1]$ is a forgetting factor

MUK Simulation Example - Fixed Channel



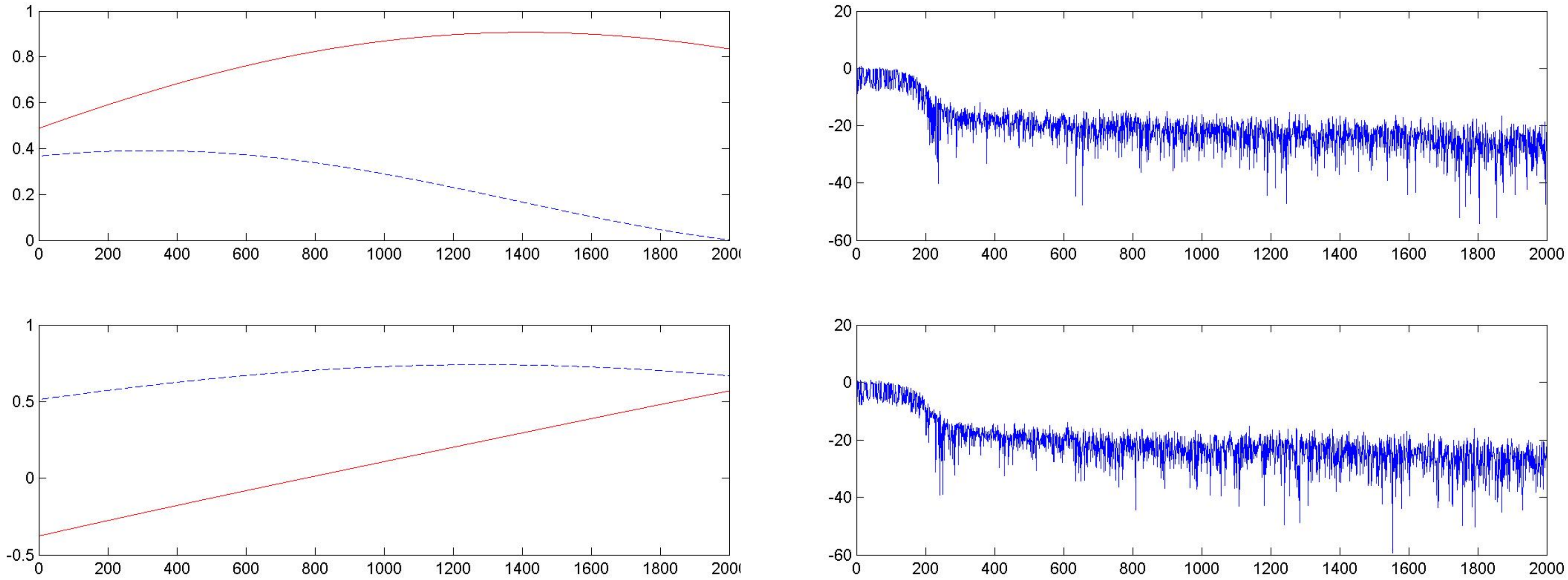
- SNR = 20 dB, $\mu = 0.02$

CMA Simulation Example - Fixed Channel



- SNR = 20 dB, $\mu = 0.02$

CMA Simulation Example - Slow-Fading Channel



- SNR = 30 dB, $\mu = 0.05$, $f_D T = 2 \times 10^{-4}$

Conclusions

- STC useful for increasing capacity in wireless communications systems
- BLAST architecture - MUK requires unitary channels
- STBC - coding structure ensures channel matrix is orthogonal
- Challenges:
  - increased convergence speed
  - time-varying channels
  - more TX than RX antennas