APPLICATION OF ICA TO BLIND IDENTIFICATION AND EQUALIZATION OF DIGITAL COMMUNICATION CHANNELS

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OUTLINE

- PROBLEM & SIGNAL MODEL
- APPLICATION OF ICA
  - joint space-time equalization
  - space equalization
- SPACE EQUALIZATION: COMPARISON
- FEATURES
  - non-Gaussian noise
  - ill-conditioned channels
- ICA-AIDED DETECTION
- ISSUES

PROBLEM & SIGNAL MODEL (1/3)

- Future wireless multimedia communication networks
  - high spectral efficiency
  - non-orthogonal multiple access (e.g., CDMA)
  - ISI + CCI $\rightarrow$ **space-time equalization**
- Spatio-temporal diversity $\rightarrow$ **MIMO** model
- **Blind** processing: no training sequences $\rightarrow$ spectrally efficient
- Received signal:

$$x_n^{(i)} = \sum_{k=1}^{K} \sum_{m=0}^{M} h_{k,m}^{(i)} s_{k,n-m} + w_n^{(i)}, \qquad i = 1, \ldots, L.$$

PROBLEM & SIGNAL MODEL (2/3)

$$\mathbf{x}_n^{(i)} = [x_n^{(i)}, \ldots, x_{n-N+1}^{(i)}]^{\mathrm{T}}, \qquad \mathbf{x}_n = [\mathbf{x}_n^{(1)\,\mathrm{T}}, \ldots, \mathbf{x}_n^{(L)\,\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^P$$

$$\mathbf{s}_{k,n} = [s_{k,n}, \ldots, s_{k,n-C+1}]^{\mathrm{T}}, \qquad \mathbf{s}_n = [\mathbf{s}_{1,n}^{\mathrm{T}}, \ldots, \mathbf{s}_{K,n}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^D$$

$$P \triangleq LN, \qquad C \triangleq M + N, \qquad D \triangleq KC$$

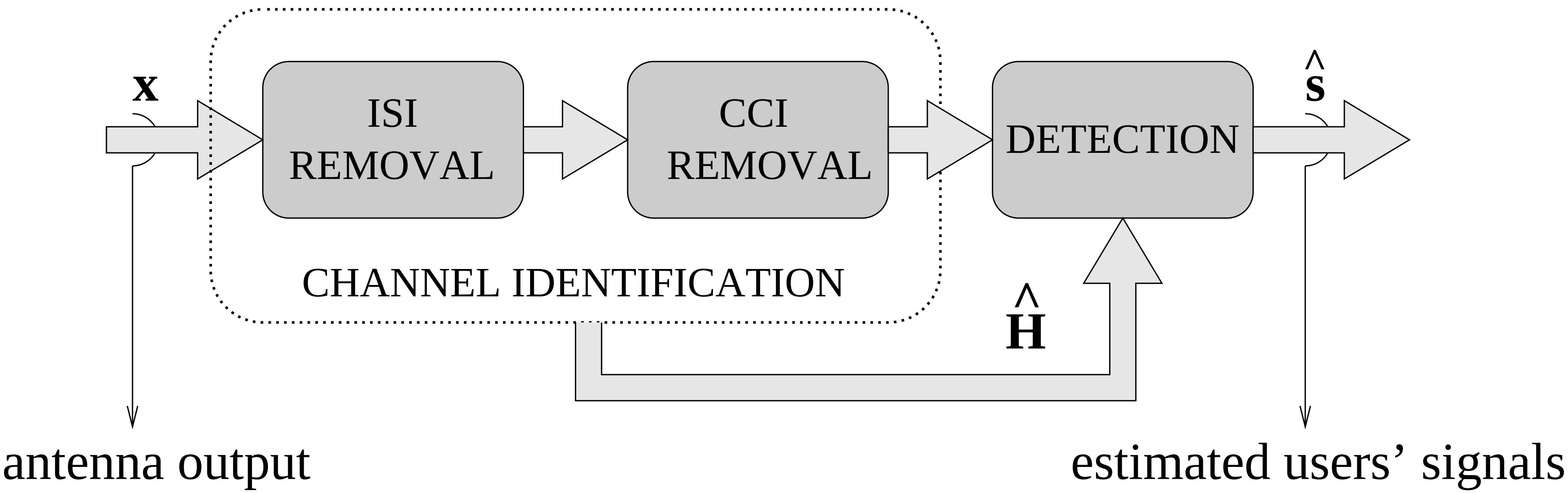
$$\mathbf{H}_{k,N}^{(i)} = \begin{bmatrix} h_{k,0}^{(i)} & \ldots & h_{k,M}^{(i)} & \ldots & \ldots & 0 \\ 0 & h_{k,0}^{(i)} & \ldots & h_{k,M}^{(i)} & \ldots & 0 \\ \vdots & & \ddots & & & \vdots \\ 0 & \ldots & 0 & h_{k,0}^{(i)} & \ldots & h_{k,M}^{(i)} \end{bmatrix} \in \mathbb{C}^{N \times C}$$

$$\mathbf{H}_{k,N} = [\mathbf{H}_{k,N}^{(1)\,\mathrm{T}}, \ldots, \mathbf{H}_{k,N}^{(L)\,\mathrm{T}}]^{\mathrm{T}}, \qquad \mathbf{H}_N = [\mathbf{H}_{1,N}, \ldots, \mathbf{H}_{K,N}] \in \mathbb{C}^{P \times D}$$

$$\boxed{\mathbf{x}_n = \mathbf{H}_N\, \mathbf{s}_n + \mathbf{w}_n}$$

PROBLEM & SIGNAL MODEL (3/3)

- **Objective:** From $\mathbf{x}_n$
  - recover $\mathbf{s}_n \Rightarrow$ blind time equalization (ISI cancellation)
    blind space equalization (CCI cancellation)
  - recover $\mathbf{H}_N \Rightarrow$ blind channel identification



ICA — JOINT SPACE-TIME EQUALIZATION

- **i.i.d. case** $\rightarrow$ components of $\mathbf{s}_n$: statistically independent
  - users' signals: spatial independence
  - data symbols: time independence

- Direct application of ICA $\rightarrow$ **joint space-time equalization**

- Ordering indeterminacy $\rightarrow$ algorithm for channel identification

- $K(M+N)$ source components $\rightarrow$ computationally efficient?

ICA — SPACE EQUALIZATION

- Direct extensions of single-user BIE methods:

$$\hat{\mathbf{H}} = \mathbf{H}(\mathbf{A}^{-1} \otimes \mathbf{I}_C), \quad \mathbf{A} \in \mathbb{C}^{K \times K}$$

- Equalized outputs (ZF): $\mathbf{z}_n = \hat{\mathbf{H}}^{\dagger}\mathbf{x}_n = \hat{\mathbf{H}}^{\dagger}\mathbf{H}\mathbf{s}_n = (\mathbf{A} \otimes \mathbf{I}_C)\,\mathbf{s}_n$
  - define $\mathbf{z} = [\mathbf{z}_n(1),\, \mathbf{z}_n(C+1),\, \ldots,\, \mathbf{z}_n\big((K-1)C+1\big)]^{\mathrm{T}}$

$$\boxed{\mathbf{z} = \mathbf{A}\mathbf{s}}$$

  - ISI elimination
  - CCI-only cancellation problem $\rightarrow$ **space equalization**

- Exploitation of users' statistical independence $\rightarrow$ ICA

- Only K source components

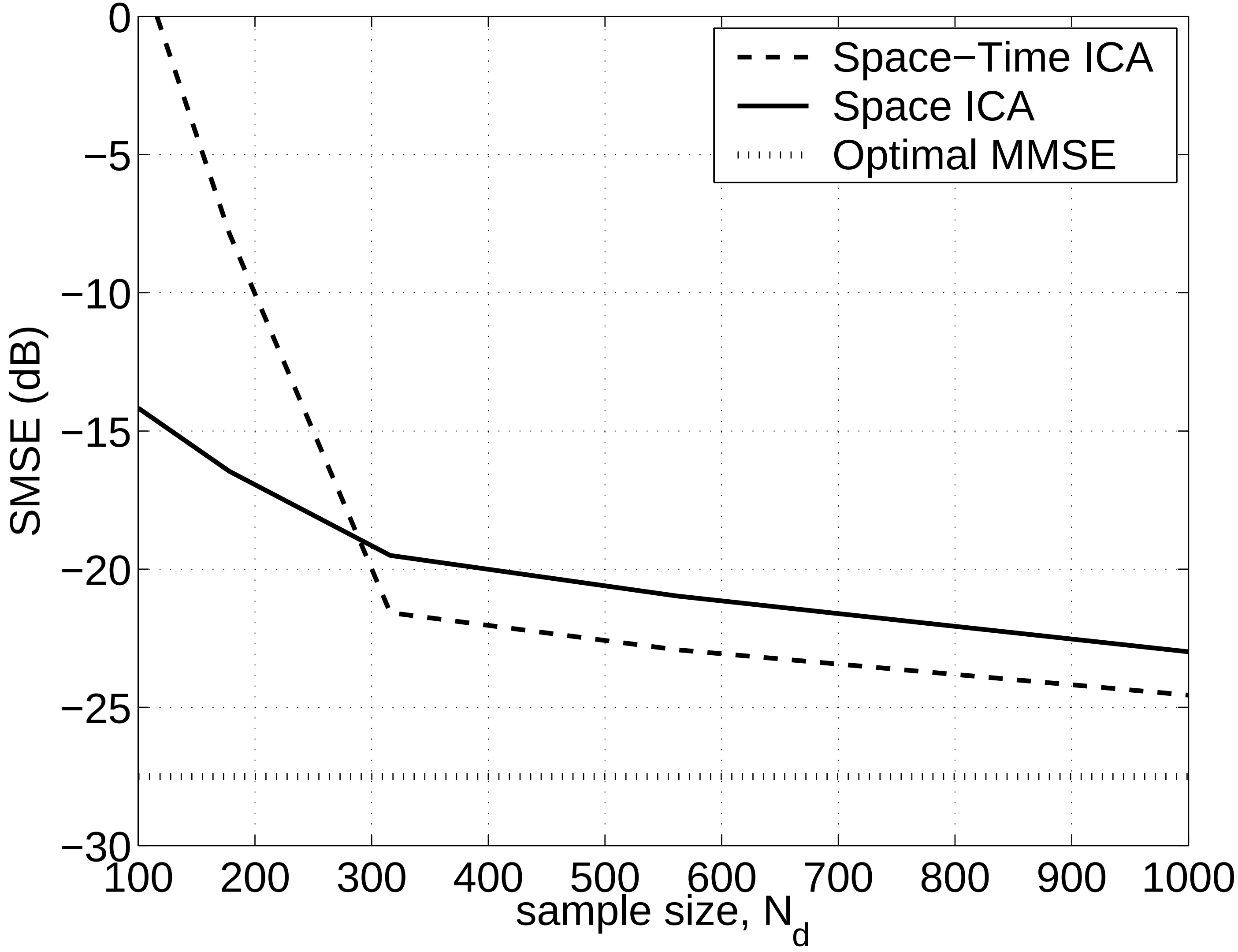
ICA — SPACE-TIME vs. SPACE EQUALIZATION

Space-Time	Space
JADE	[TON94]
	JADE

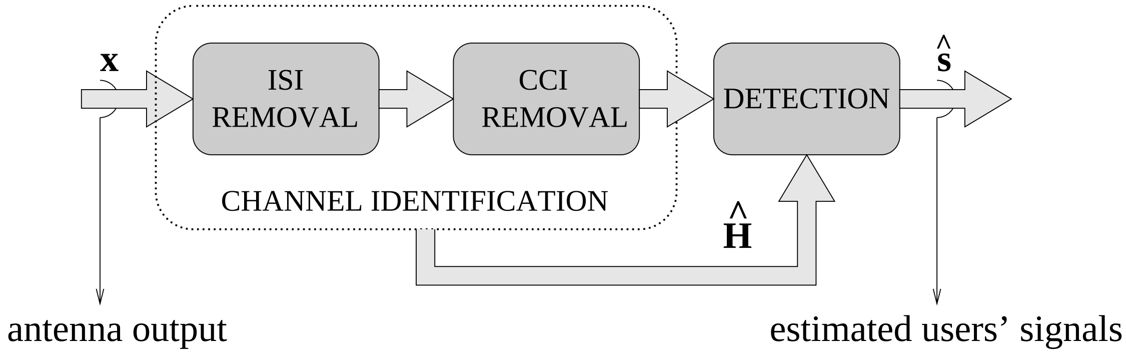
Number of users, K	2
Modulation	QPSK
Receive antenna	1
Oversampling factor, L	6
Smoothing factor, N	2
Channel delay spread, $M+1$	3 symbol periods
Channel matrix, $\mathbf{H}$	12×8; cond$\# \approx 5$

ICA — SPACE-TIME vs. SPACE EQUALIZATION

- SNR $= \text{trace}(\mathbf{H}\mathbf{H}^{\mathrm{H}})/(\sigma^2 P)$; SNR $= 30$ dB



SPACE EQUALIZATION: COMPARISON (1/4)



- **ISI cancellation**
  - subspace approach
- **CCI cancellation**
  - FA-based method: ILSF
  - ICA-based method: FastICA
- **Detection**
  - subspace MMSE
- **Performance**
  - signal mean square error (SMSE)

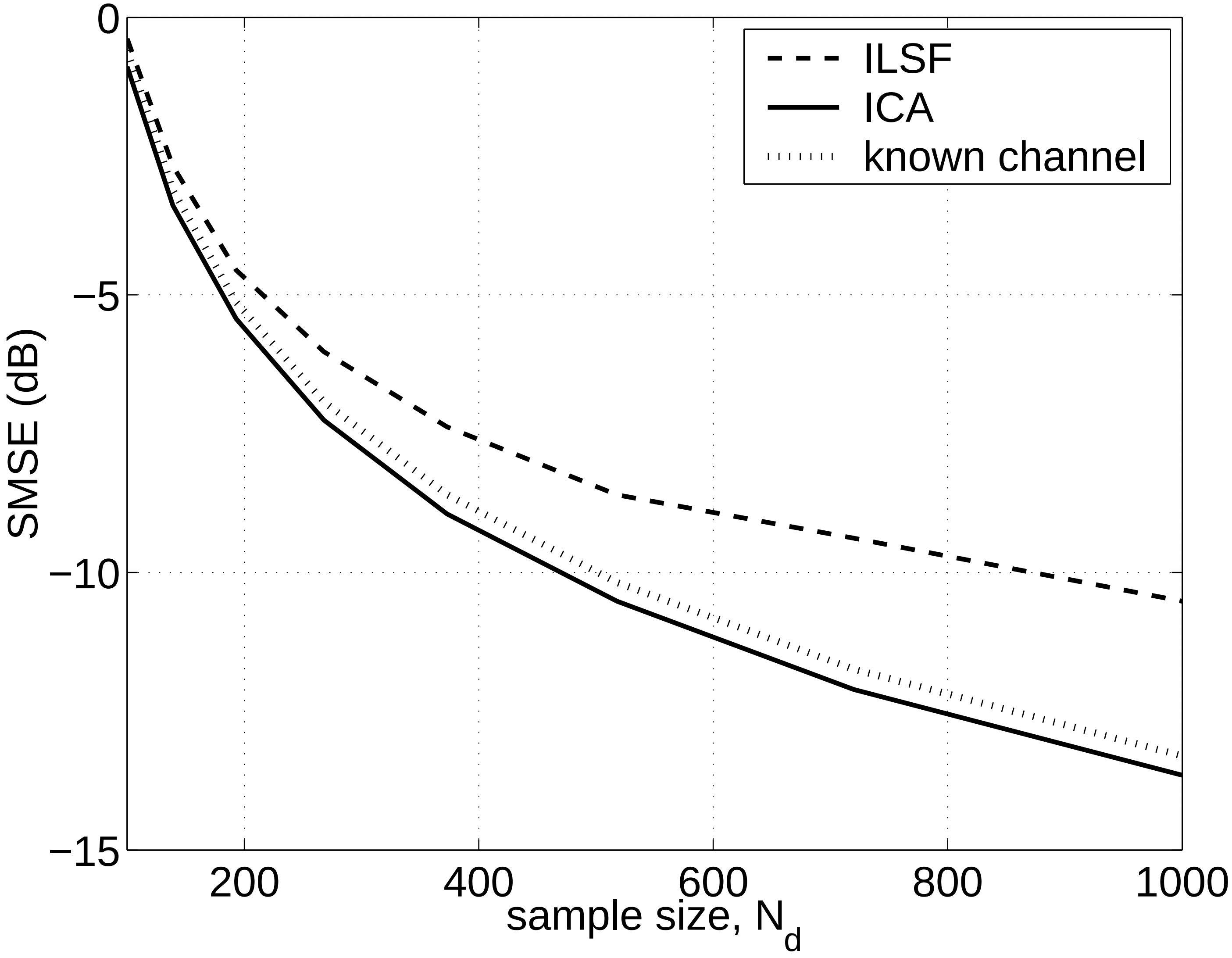
SPACE EQUALIZATION: COMPARISON (2/4)

- Simulated multiuser communication system:

Number of users, K	5
Modulation	QPSK
Receive antenna	1
Oversampling factor, L	10
Smoothing factor, N	5
Channel delay spread, $M+1$	5 symbol periods
Channel matrix, $\mathbf{H}$	50×45; cond# ≈ 100

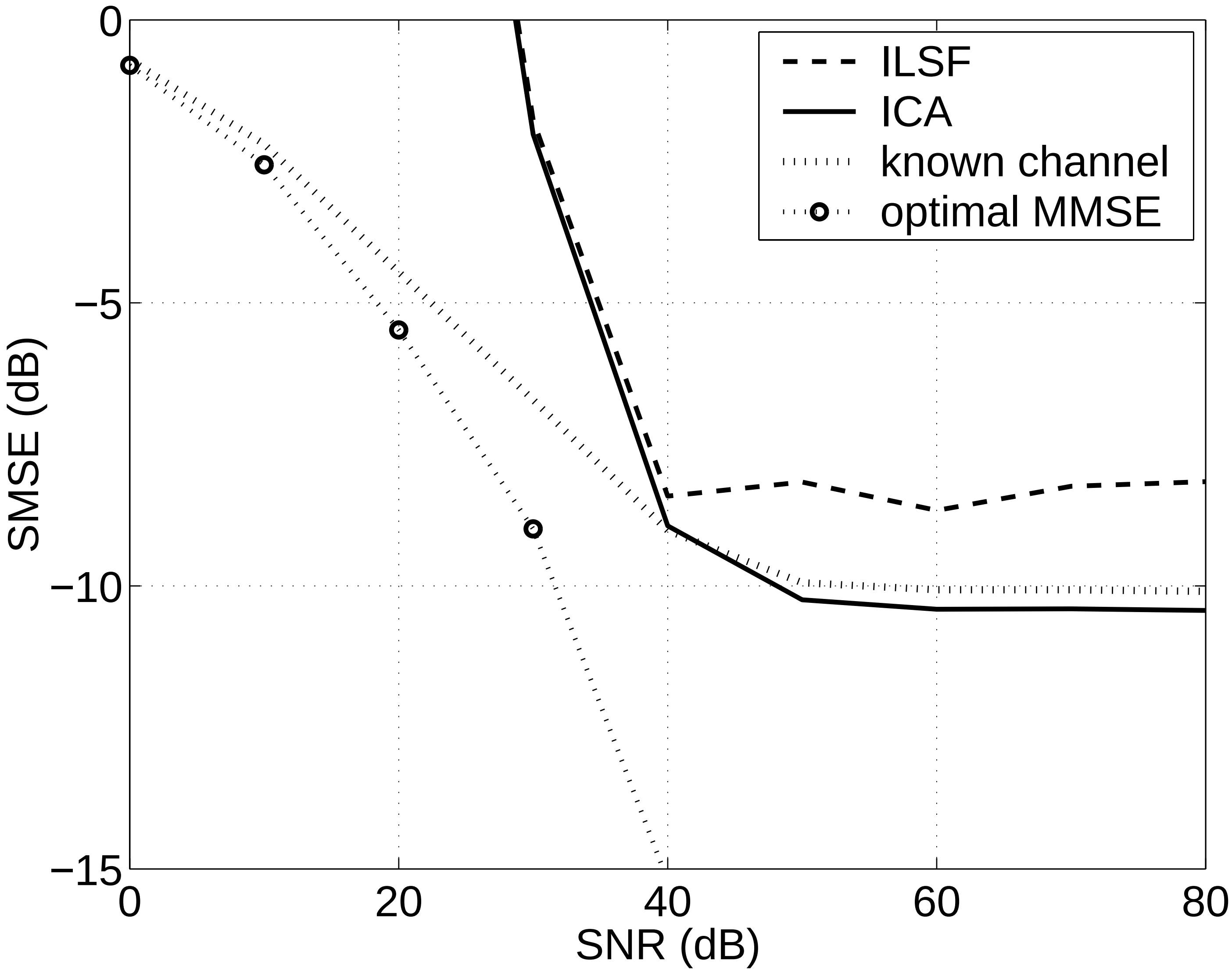
SPACE EQUALIZATION: COMPARISON (3/4)

- Noiseless case



SPACE EQUALIZATION: COMPARISON (4/4)

- $N_d = 500$ symbol periods; SNR = trace$(\mathbf{HH}^{\mathrm{H}})/(\sigma^2 P)$



FEATURES — NON-GAUSSIAN NOISE (1/2)

- ZF detection after ISI cancellation:

$$\mathbf{z}_n = \hat{\mathbf{H}}^\dagger \mathbf{x}_n = \hat{\mathbf{H}}^\dagger \mathbf{H}\mathbf{s}_n + \underbrace{\hat{\mathbf{H}}^\dagger \mathbf{w}_n}_{\text{equalized noise}}$$

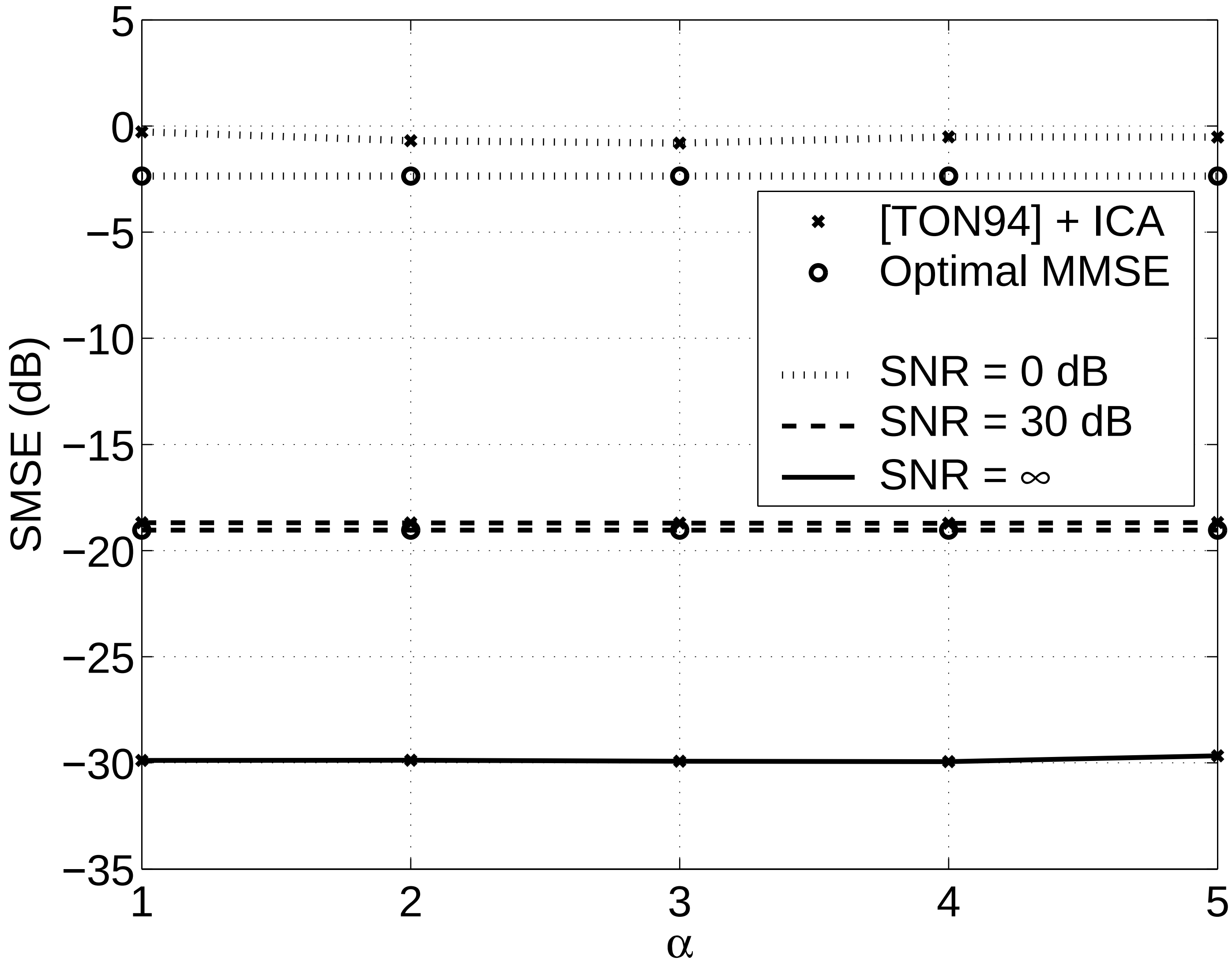
- Central Limit Theorem $\rightarrow$ equalized noise $\approx$ Gaussian

- Test: CGGD noise $p(w) \propto \exp(-|w|^\alpha)$

Number of users, K	3
Modulation	16-QAM
Oversampling factor, L	12
Smoothing factor, N	2
Channel delay spread, $M+1$	6 symbol periods
Channel matrix, $\mathbf{H}$	24×21; cond$\# \approx 30$

FEATURES — NON-GAUSSIAN NOISE (2/2)

- $N_d = 10^4$ symbol periods; SNR $=$ trace$(\mathbf{H}\mathbf{H}^{\mathrm{H}})/(\sigma^2 P)$



FEATURES — ILL-CONDITIONED CHANNELS

- Equivariance of ICA/BSS methods $\rightarrow$ uniform performance
- Robust performance for ill-conditioned channels
- Only expected in the noiseless case (high SNR)
- Test: channel matrix with varying condition number

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Modulation	16-QAM
Oversampling factor, L	12
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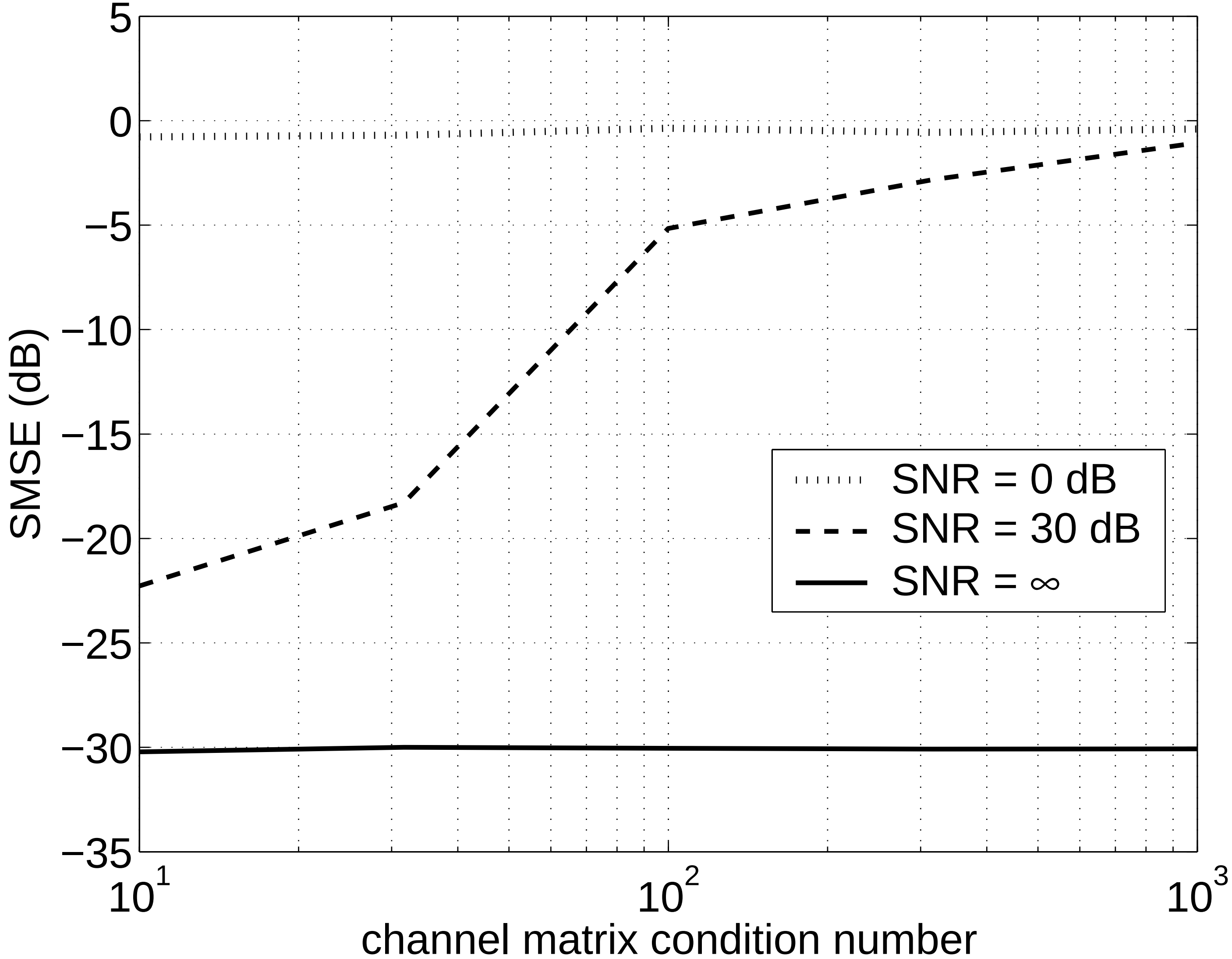
Channel delay spread, $M+1$	6 symbol periods
Channel matrix, $\mathbf{H}$	24×21; cond# ≈ 30

FEATURES — ILL-CONDITIONED CHANNELS

- $N_d = 10^4$ symbol periods; SNR $=$ trace$(\mathbf{H}\mathbf{H}^{\mathrm{H}})/(\sigma^2 P)$



ICA-AIDED DETECTION (1/4)

- Statistical independence can be further exploited at detection

- Linear **MMSE** detector:

$$\hat{\mathbf{s}}_n = \mathbf{G}^{\mathrm{H}}\mathbf{x}_n, \quad \mathbf{G} = \mathbf{R}_x^{-1}\hat{\mathbf{H}}$$

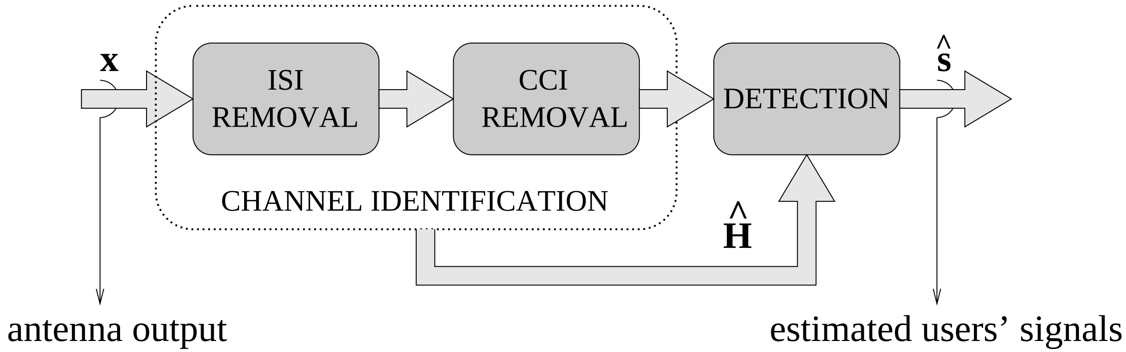
- **MMSE-ICA** detector:

$$\mathbf{G} \rightarrow \boxed{\text{ICA}} \rightarrow \tilde{\mathbf{G}}$$

- **Simplified MMSE-ICA** detector:

Redundancy of MIMO model $\rightarrow \mathbf{G}^{(K)} = \mathbf{R}_x^{-1}\hat{\mathbf{H}}^{(K)}$

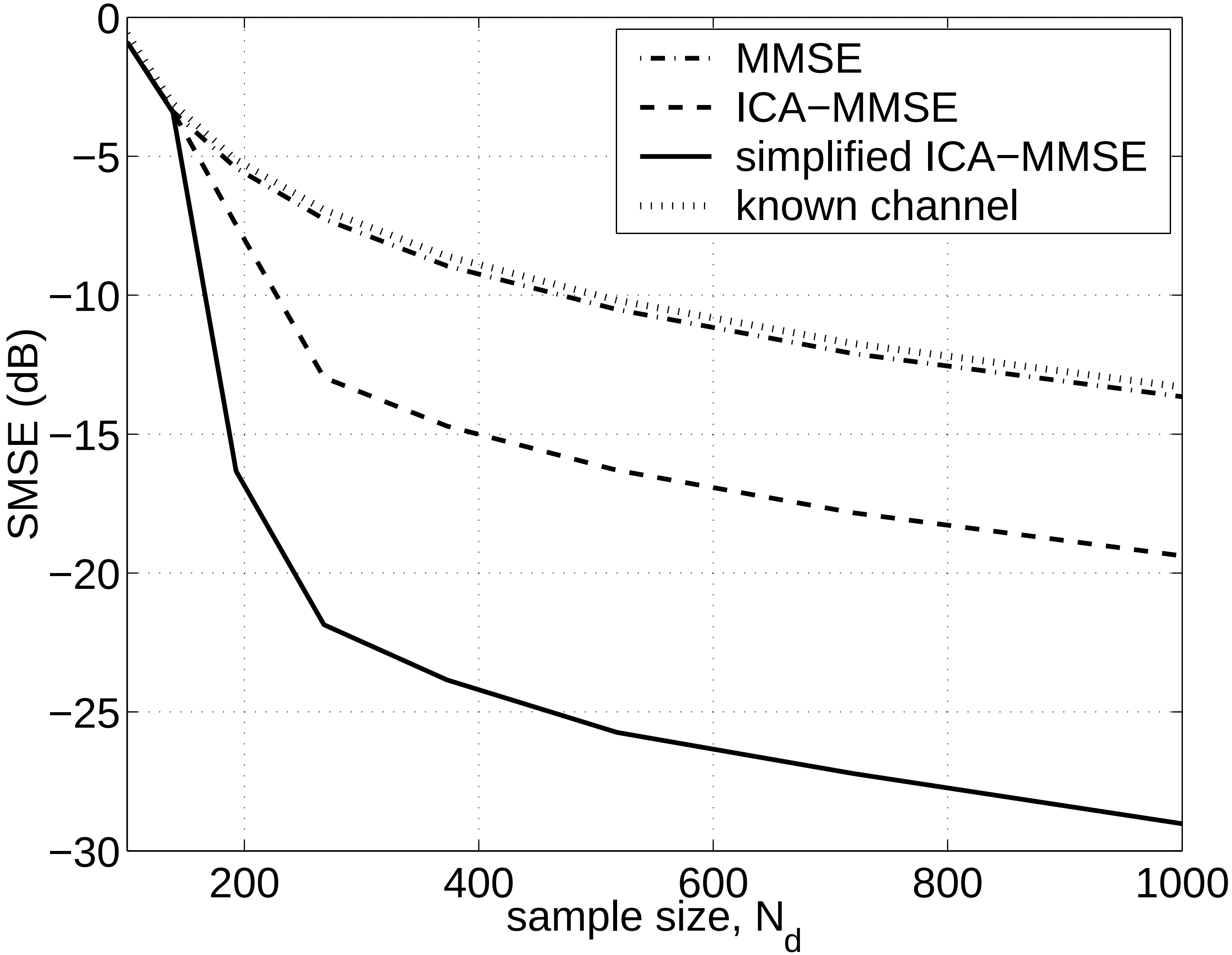
ICA-AIDED DETECTION (2/4)



- **ISI cancellation**
  - subspace approach
- **CCI cancellation**
  - ICA-based method: FastICA
- **Detection**
  - subspace MMSE
  - ICA-aided subspace MMSE
  - simplified ICA-aided subspace MMSE
- **Performance**
  - signal mean square error (SMSE)

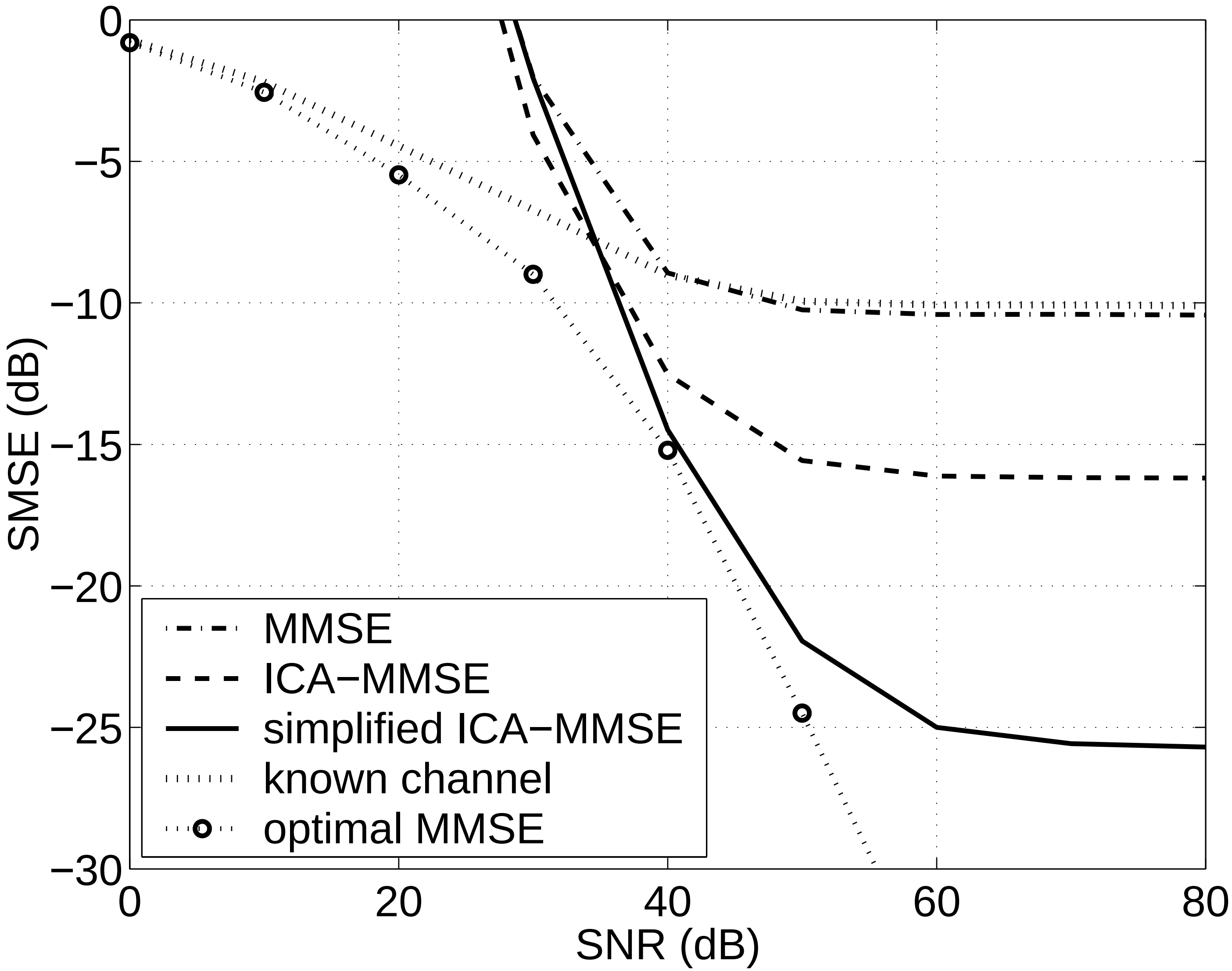
ICA-AIDED DETECTION (3/4)

- Noiseless case



ICA-AIDED DETECTION (4/4)

- $N_d = 500$ symbol periods; SNR = trace$(\mathbf{H}\mathbf{H}^{\mathrm{H}})/(\sigma^2 P)$



ISSUES

- Computational complexity $\rightarrow$ choice of ICA method
- Estimation of signal subspace dimensions
- Channels with different delay spreads
- BSS of convolutive linear mixtures
- Non-stationary environments $\rightarrow$ fast adaptive implementations
- Advanced multiple access techniques (OFDM, CDMA, ...)
- More realistic MIMO models

SUMMARY

- BIE of multiuser digital communication channels
- Users' statistical independence $\rightarrow$ **ICA**
  - blind $\rightarrow$ BW efficient
  - enhanced CCI cancellation
  - blind to signal modulation
  - robust to non-Gaussian noise
  - robust to ill-conditioned channels (high SNR)
  - remarkably improved symbol detection
- Further research