Flip Phenomena and Co-existing Attractors in an Incremental Actuator

Josh Reiss[1], Bruno Robert[2], Francois Alin[2], Mark Sandler[1]

[1]Dept. of Electronic Engineering
Queen Mary, University of London
Mile End Road, London E14NS, U.K.
josh.reiss,mark.sandler@elec.qmul.ac.uk

[2]Laboratoire des Systèmes de Décision
et de Télécommunications
UFR Sciences Exactes et Naturelles
Moulin de la Housse, BP 1039
51687 Reims Cedex 2, France
francois.alin,bruno.robert@univ-reims.fr

Abstract– **The stepper motor converts electrical pulses into mechanical movements. The shaft of the motor rotates in discrete increments when command pulses are applied in the proper sequence. The dynamic behavior of this motor is of great importance, since instabilities lead to velocity fluctuations that are unacceptable in many applications. It is a major limitation to development of a high performance open-loop step motor system. This work represents a careful analysis of observed phenomena when this motor is operated within the chaotic regime. We report on several phenomena that have been observed in the experimental system. These phenomena, which include the alternating of the dynamics between two experimentally observable variables and the existence of co-existing attractors, greatly influence the types of control schemes that can or should be applied.**

1 Overview

The stepper motor dynamics have been analysed in previous works[1-5]. In this work, we identify some unusual behaviors that are not predicted by simulation.
The step motor is typically operated below 20 Hz because the dynamics in that range are known to produce a simple stable fixed point. We seek to determine the behaviour of the electric step motor over a range of high frequency dynamics
By characterising the high frequency dynamics, one may be able to determine the appropriate control schemes required in order to create regular, periodic motion throughout this frequency range. It is also hoped that we will be able to determine how accurately the motor dynamics agrees with the predicted dynamics from the model. If the agreement is strong, then we will be able to use the known dynamics of the model as a predictor for the motor, and we can exploit this knowledge in control schemes.
The data consisted of 4,000 data sets over the range 40.01 to 80.00 Hz, in increments of 0.01 Hz. Each data set consisted of 100,352 two dimensional data points, where current I_a and current I_b were acquired and scaled to values in the range [-2^{15}, $+2^{15}$]. Points were sampled at the applied frequency, so that the data represents points from a stroboscopic, or Poincare, section. Known noise in the system was due to jitter in the sampling and quantisation errors (clipping and low bit approximation).

The dynamics as a function of frequency were studied using a variety of methods, of which partial results for the following methods are presented within;

1. Mean values of currents as a function of frequency.
2. Waveform plots.
3. Poincare section plots at relevant frequencies.
4. Detailed analysis of select frequencies.

Analysis of the mean should isolate frequencies at which there is an abrupt change in the dynamics, as well as showing a gradual drift in the mean output currents. Unfortunately, plots of statistical quantities with respect to applied frequency do not inspect the individual dynamics at each frequency. For instance, this does not act as a satisfactory indicator of whether the dynamics are chaotic or quasiperiodic. For this we need visual analysis where the frequency is constant for each plot.
Generating and investigating thousands of plots (for each of the 4,000 frequencies) is costly both in terms of processing time and in terms of the researcher's time. Thus specialised software was designed to create these plots in batch mode. Waveform plots were generated for each frequency. Then Poincare section plots were created for all frequencies where the waveform plots identified unusual or complex behaviour. Where the dynamics were still not effectively quantified, these frequencies were isolated and investigated individually using a variety of methods.
Due to the very large amount of data (over 800 million integer values), many important phenomena may have been overlooked. However, the qualitative analysis performed should allow us to classify most, if not all, of the observed behaviour.

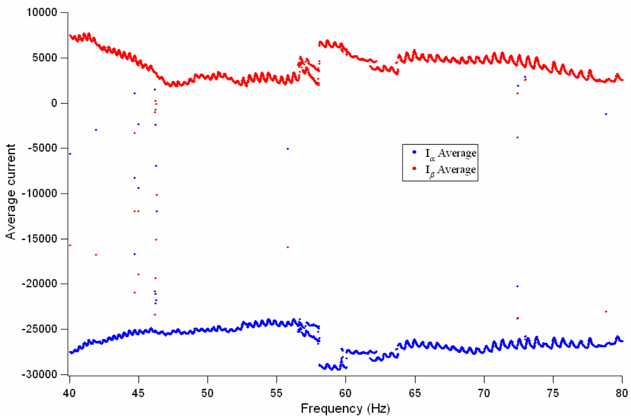

Figure 1. Average value as a function of frequency.

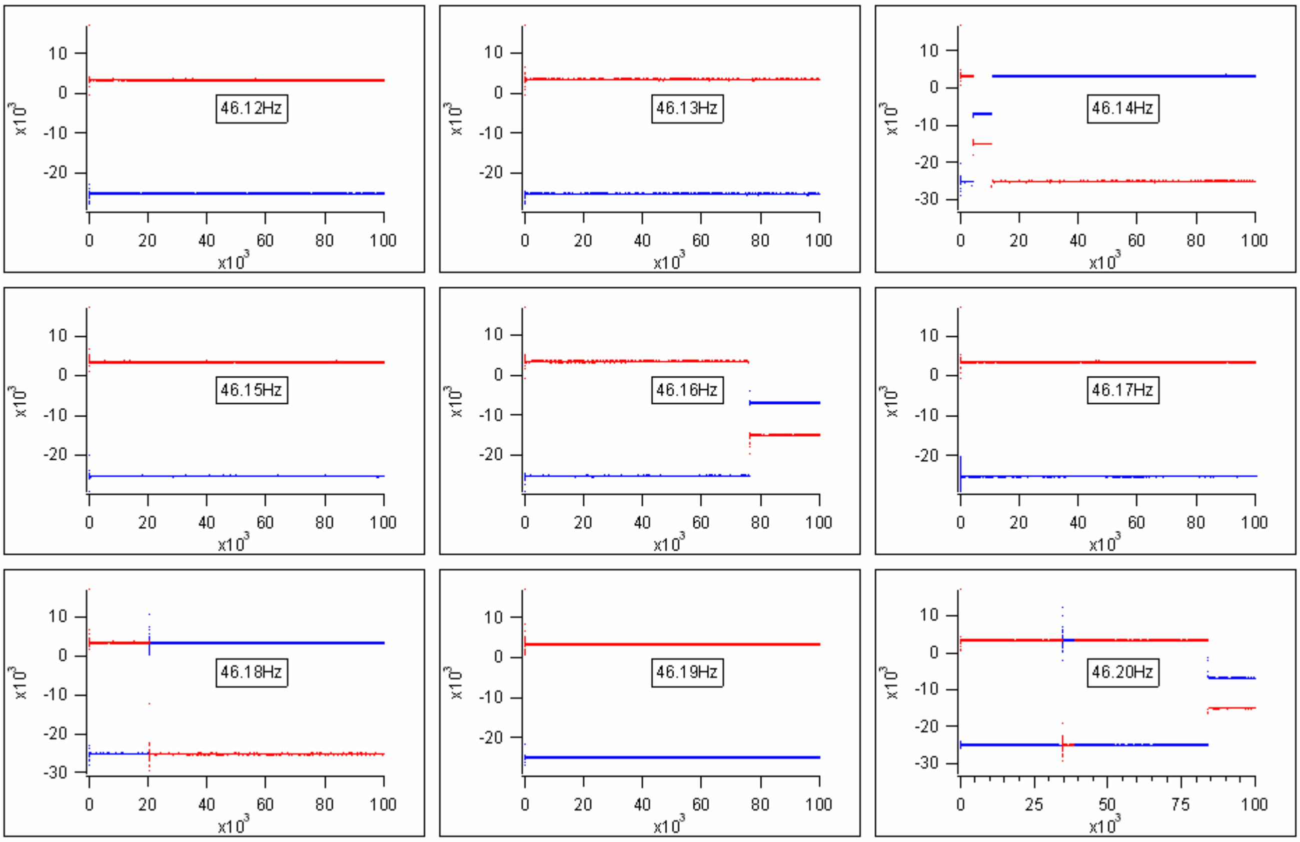

Figure 2. Waveform plots in a region where unusual behavior has been observed.

The full extent of the analysis is beyond the scope of this work. Here, we present a few examples of the rich and varied behaviors exhibited by the stepper motor throughout the range 40-80kHz.

2 Global Behavior

Figure 1 provides a plot of the average value of each current as a function of frequency. At each frequency, the average is taken over the entire data set. Discontinuities in this plot (i.e., a mean value at a given frequency differs greatly from neighboring values) occur at only 23 out of 4,000 frequency values. Many phenomena might account for this behaviour, but it should be noted that this behaviour is extremely rare. It should also be noted that although there is some symmetry between the I_a and I_b values, they differ greatly. Furthermore, we can notice discontinuous behaviour in several places, most notably around 58.00.

Waveform analysis provides a more precise description of the dynamics. Most importantly, this can explain the unusual behaviour occurring at select frequencies. Typical waveforms that demonstrate the phenomena identified as being atypical from the mean and standard deviation plots are depicted in Figure 2, along with waveforms at nearby frequencies which *do not* exhibit this behaviour. All exhibit period 1 behaviour, but there exists occasional flips of I_a and I_b. In addition, an entirely different period 1 orbit is observed. This ‘other’ period 1 orbit is observed at the first measured frequency, 40.01 Hz. Thus we cannot make the assumption that the unusual results observed at the initial frequency are a by-product of transient behaviour.

Analysis of those frequencies where the unusual behavior has not yet been explained was achieved through the use of Poincare sections? For this, I_b was plotted against I_a. The main achievement of such a technique, as demonstrated in the following sections was to distinguish chaotic from quasiperiodic dynamics. Here the distinction was made qualitatively, since a proper validation of chaotic dynamics is arduous and subject to noise and nonstationarity.

3 Dynamics at 49 Hz

This data set provides ample opportunity to study how the motor can switch between different dynamics. For the purposes of control, it is also important to study the behavior at this frequency, since we may wish to control quasiperiodic behavior and make it periodic.

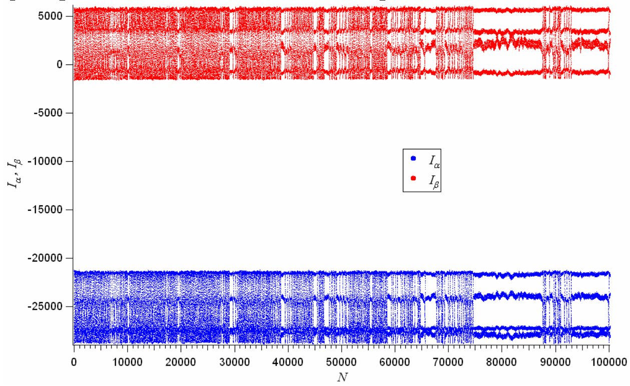

Figure 3. Waveform plots (current versus point number *N*) at 49Hz.

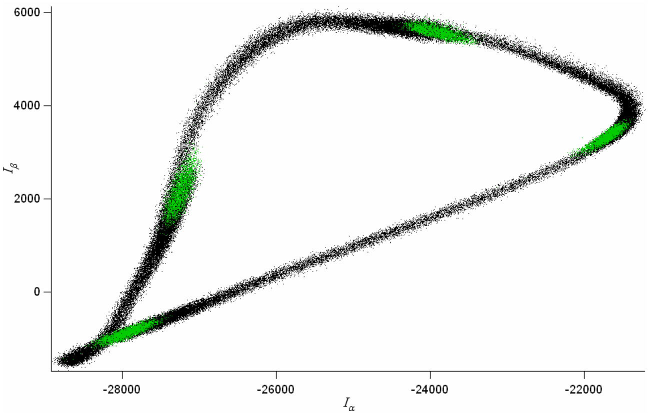

Figure 4. Poincare section plot of I_b versus I_a. Initial transients have been removed and points 74,700 to 86,700 have been plotted in green.

Frequent transitions between quasiperiodic behavior and period 4 behavior are depicted in Figure 3. The length of time spent in each regime varies greatly. There is a window of at least 12,000 points (74,700 to 86,700), or more than 215 seconds, where period 4 behavior is maintained. At other times, especially towards the beginning of the data set, period 4 behaviour is maintained for less than a couple of hundred iterations.

The Poincare section in Figure 4 offers a more descriptive view of the dynamics. Points 74,700 to 86,700 are colored green in order to distinguish them from the remainder of the data. The complex behavior in the waveform is revealed to be quasiperiodicity. Within this quasiperiodic orbit is a co-existing, semi-stable period 4 orbit. The time required to leave the orbit can be quite significant, and the rate at which points diverge away from the orbit does not appear to be exponential.

The entire range from 48.80 to 49.05Hz exhibits the existence of periodic orbits within quasiperiodic orbits. It is also repeated at different frequencies, although the period of the periodic orbit need not be the same. As the frequency is changed, either periodic orbits are becoming more stable, or aperiodic orbits are appearing to be low period limit cycles. Thus the quasiperiodic motion, with the addition of some noise, may remain for long periods of time within a periodic orbit.

4 Dynamics at 55.75 Hz

One of the most unusual observed dynamics occurs at 55.75Hz. A period 7 window occurs from about 55.65 Hz to 55.95Hz. However, the dynamics at 55.75 Hz bears no relation to the dynamics at any of the surrounding frequencies. A waveform plot (not depicted, since it is just straight lines) reveals this to be periodic and stable, with no significant transient or intermittent behavior where it might revert back to the period 7 orbit. The Poincare section is compared with the Poincare section at a nearby frequency, 55.74Hz, in Figure 5. One can see that a period 7 orbit also exists at 55.75Hz, but there is no overlap between the two Poincare sections and no obvious symmetry. All other frequencies in this period 7 window have data residing in the vicinity of the Poincare section at 55.74Hz. This is evidence of a co-existing attractor, but the mechanism for migration between the attractors is unknown.

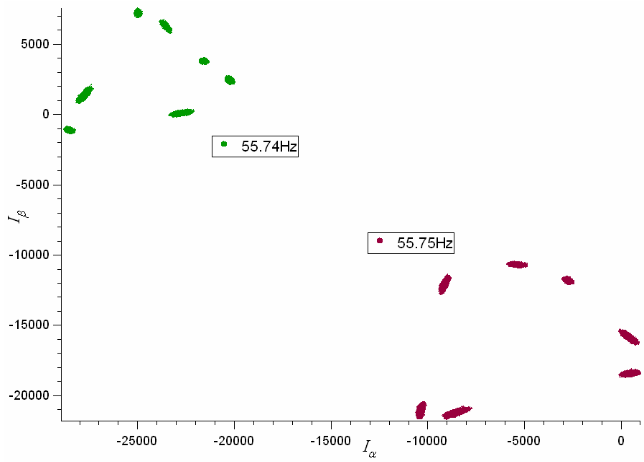

Figure 5. Poincare section plots of I_b versus I_a at nearby frequencies with initial transients removed.

5 Dynamics at 58 Hz

At 58Hz, we observe three distinguishable forms of behavior. An examination of the waveforms (Figure 6) reveals not just possibly chaotic and periodic behavior, but also intermittent behavior within the periodic regime. For instance, there is a clear "burst" that begins shortly after 6950 and ends shortly before 12850. Bursts are repeated several times throughout the data. In other portions after the initial regime (up to 6950), there appears to be period 2 behavior in I_a and period 1 behavior in I_b.

This is more clearly depicted in Figure 7, which provides a Poincare section plot of I_b versus I_a. Different points in time are plotted in different colors in order to associate regions of the waveform with regions in the section. The initial motion is chaotic. The dominant periodic regimes represent a period 2 orbit. The less frequent periodic regime is a period 12 orbit that surrounds the period 2 orbit. The three attractors are near enough to provide an opportunity for transition between states due to noise and/or parameter drift. The remaining points in the data set represent transitions between the periodic orbits.

This period 12 orbit is only seen again at 57.90Hz. Other frequencies from 57.75 to 58.25Hz only show four types of behavior; period 1 in I_a and period 2 in I_b, period 2 in I_a and period 1 in I_b, chaos and transient chaos. This frequency range represents the onset of chaotic motion. Thus this unusual behavior may be a sign of instability in the dynamics, i.e., an indicator of possible chaos.

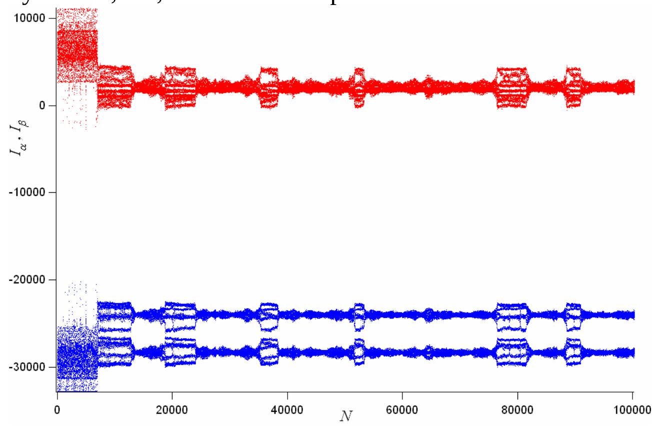

Figure 6. Waveform plot at 58Hz.

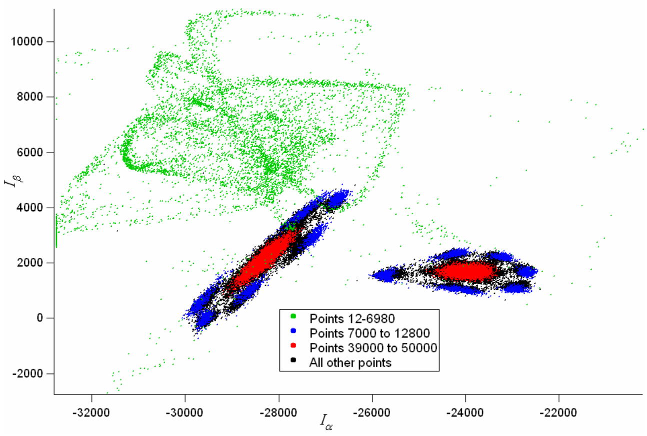

Figure 7. Poincare section of I_b versus I_a. Different times are in different colors to associate regions of the waveform with regions of the section.

6 Conclusion

We are now ready to give a full characterisation of the observed dynamics. This is provided by Table 1. The most typical behaviour is period 1, but there are also other low period co-existing orbits that appear at a large number of frequencies. Furthermore, the complex dynamics that are observed by a variety of methods are found by the use of Poincare sections to be mostly quasiperiodic behaviour. Chaotic behaviour occurs only over a narrow range of frequencies, and is often observed only as intermittent or transient behaviour. In addition, a frequently observed transitional phenomenon is the existence of different periods for I_a and I_b. In the region just prior to full chaotic motion, 56.51-58.06 Hz, the system frequently alternates between I_a maintaining period 1 motion and I_b having period 2 motion, and vice-versa. Perhaps this is a mechanism which leads to chaotic behaviour.

The dynamic behaviour of the step motor has been investigated over the range 40.01 to 80.00 Hz. This was achieved through the analysis of 4,000 data sets, each consisting of over 100,000 I_a and I_b sampled current values. The behaviour can be characterised by periodicities, transients, intermittency, quasiperiodicity and chaos. Furthermore, the transitions between these behaviours was also observed. These transitions are gradual, and thus there is no evidence of sudden crises. Instead, as the frequency is changed, there is a gradual change in the stability of orbits.

In each of the main dynamic classifications that were observed; periodic, chaotic, quasiperiodic, intermittent and transient behaviour, there was also evidence of co-existing, low period limit cycles. Thus it is our belief that control schemes should take advantage of this. The dominant control methods for systems that exhibit chaotic motion are based on stabilising unstable periodic orbits. Instead, we propose that control should be based on entrainment and migration – find the coexisting periodic orbit, move there and stay there. The difficulty with this proposal is that the coexisting attractor may not be observed often. This can be solved if the model accurately describes the dynamics. In which case we can determine the location of unobserved coexisting attractors, and apply a control method in order to move the dynamics towards them.

Table 1. Full characterisation of dynamics observed in the motor.

Frequency (Hz)	**Behavior**
40.01	Atypical period 1
40.02-41.86	Period 1
41.87	Period 1 with flip of I_a and I_b
41.88-44.65	Period 1
44.66-70	Two coexisting period 1s, occasional flips
44.71-44.93	Period 1
44.94-44.98	Two coexisting period 1s, occasional flips
44.99-46.13	Period 1
46.14-46.28	Two coexisting period 1s, occasional flips
46.29-47.71	Period 1
47.72-48.00	Transition to quasiperiodic behavior
48.01-51.15	Quasiperiodic, occasional periodic on same orbit
48.85-48.92	Period 4, occasional quasiperiodic behavior
48.93-51.15	Quasiperiodic, rare periodic on same orbit
49.59-49.74	Period 8 or 9 plus quasiperiodic behavior
49.75-50.07	Quasiperiodic, occasional periodic on same orbit
50.08-50.19	High period (8 or 9?) plus quasiperiodic
50.20-51.15	Quasiperiodic, frequent periodic on same orbit
51.16-52.75	Period 3, increasing transient
52.76-54.61	Quasiperiodic, occasional periodic on same orbit
54.62-54.81	Period 5
54.82-55.67	Quasiperiodic with period 7 on same orbit.
55.68-55.74	Period 7
55.75	Enters different period 7 attractor
55.76-55.96	Period 7 (sometimes appears period 5 or 6)
55.97-56.27	Quasiperiodic
56.28-56.50	Flips between quasiperiodic and periodic
56.51-58.06	Noisy periodic I_a period 1 or 2, I_b period 1 or 2, Increasing chaotic transient
58.07-59.63	Chaos
59.64-60.02	Chaos interspersed with period 2.
60.03	Period 2
60.04	Intermittent period 2 and chaos
60.05-63.61	Period 2
63.62-64.24	Period 2 in I_a, Period 1or period2 in I_b
64.25-65.99	Period 2
66.00-68.89	Period 2 in I_b, Period 1 in I_a
68.90-70.49	Convergence of I_b , period 2 to period 1
70.50-72.37	Period 1
72.38-72.43	Two coexisting period 1s, occasional flips
72.44-72.94	Period 1
72.95-78.98	Coexisting period 1s, occasional flips
72.99-78.78	Period 1
78.79	Period 1, I_a and I_b switch
78.80-80.00	Period 1

References

[1] L. Y. Cao and H. M. Schwartz, *Oscillation, instability and control of stepper motors*, Nonlinear Dynamics **18** (4), 383 (1999).

[2] Pera M.C., Robert B., Goeldel C., 2000, Nonlinear dynamics in electromechanical systems -application to a hybrid stepping motor. Electromotion, 7, 31-42.

[3] M. C. Pera, B. Robert, and C. Goeldel. *Quasiperiodicity and Chaos in a Step Motor.* 8th European Conference on Power Electronics and Applications, Lausanne, Switzerland1999.

[4] J. D. Reiss, F. Alin, M. Sandler et al. *A Detailed Analysis of the Nonlinear Dynamics of the Electric Step Motor.* IEEE International Conference on Industrial Technology, Bangkok, Thailand, 11-14 December 2002.

[5] F. Alin and B. Robert. *On the Limits of Chaotic Simulations by Classical Software - Applications to the Step Motor.* IEEE International Conference on Industrial Technology, Bangkok, Thailand, 2002.