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Fuzzy Impulsive Control of High Order Interpolative Lowpass Sigma Delta Modulators

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ABSTRACT

In this paper, high order interpolative sigma delta modulators (SDMs) in audio applications are considered. For these SDMs, limit cycle and divergent behaviors may be observed, especially when the inputs are overloaded. A novel fuzzy impulsive control strategy is proposed. The control law is to minimize the difference between the uncontrolled trajectory and the control trajectory, suppress the occurrence of limit cycles, and maintain the local stability of the SDMs. Examples of high order lowpass interpolative SDMs are given to illustrate the effectiveness of the proposed control strategy.

1. INTRODUCTION

Sigma delta modulation technique has been proposed and applied in analog to digital conversion for many years [1]. It is particularly popular in the past few decades because of the advance in electronic technology that makes the devices practical with low implementation cost [1]. It is particularly useful and widely applied because it possesses noise shaping characteristics that suppress the conversion error and increase the resolution of the output signals. Sigma delta modulators (SDMs) are found in many applications such as communication systems [2], consumer and professional audio processing [3], and precision measurement devices [4].

High order SDMs are preferred since they yield a higher signal-to-noise ratio (SNR) and better noise-shaping characteristics than lower order SDMs. However, high order SDMs suffer from instability problems. The existing control strategies such as [5] and [6] stabilize the loop filter by changing the effective poles of the loop filter. Since the loop filter is usually designed to have a good SNR, the SNR of the controlled SDMs will be affected or even worsen. They may also significantly distort the noise-shaping characteristics. Moreover, the parameters in the controller depend on the loop filter parameters, so a particular class of controllers may not be able to stabilize all interpolative SDMs. Furthermore, the controlled SDMs may still be unstable when the input step size is increased, or for different choices of initial conditions for the integrator states. In order to control the SDM without changing the effective poles of the loop filter, clipping method is employed. However, clipping method usually results in limit cycle. In this paper, a fuzzy impulsive control strategy is proposed.

The outline of this paper is as follows. In Section II, we introduce the notations which appear throughout this paper. In Section III, a fuzzy impulsive control law is developed. In Section IV, some simulation results of the fuzzy impulsive control strategy are presented. Finally, a conclusion is summarized in Section V.

2. NOTATIONS

The block diagram of an interpolative SDM is shown in Figure 1. The input to the SDM and the output of the loop filter are denoted as, respectively, $u(k)$ and $y(k)$. The transfer function of the loop filter is denoted as $F(z)$.

$$F(z) = \frac{\sum_{j=1}^{N} b_j z^{-j}}{\sum_{i=0}^{N} a_i z^{-i}}. \tag{1}$$

The SDM can be described by the following state space equation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}(\mathbf{u}(k) - \mathbf{s}(k)), \tag{2}$$

for $k \geq 0$, where

$$\begin{aligned} \mathbf{x}(k) &\equiv [x_1(k), \quad \cdots, \quad x_N(k)]^T \\ &\equiv [y(k-N), \quad \cdots, \quad y(k-1)]^T \end{aligned}, \tag{3}$$

$$\mathbf{u}(k) \equiv [u(k-N), \quad \cdots, \quad u(k-1)]^T, \tag{4}$$

$$\begin{aligned} \mathbf{s}(k) &\equiv [s_1(k), \quad \cdots, \quad s_N(k)]^T \\ &\equiv [Q(y(k-N)), \quad \cdots, \quad Q(y(k-1))]^T \end{aligned}, \tag{5}$$

$$\mathbf{A} \equiv \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & 1 \\ -\frac{a_N}{a_0} & \cdots & \cdots & \cdots & -\frac{a_1}{a_0} \end{bmatrix}, \tag{6}$$

and

$$\mathbf{B} \equiv \begin{bmatrix} 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 \\ \frac{b_N}{a_0} & \cdots & \cdots & \cdots & \frac{b_1}{a_0} \end{bmatrix}, \tag{7}$$

in which Q is a one-bit quantizer defined as follows,

$$Q(y) \equiv \begin{cases} 1 & y \geq 0 \\ -1 & otherwise \end{cases}. \tag{8}$$

Since the oversampling ratio of the SDM is usually very high, the input can be approximated as a step signal. Hence, we can further assume that $\mathbf{u}(k) = \bar{\mathbf{u}}$ for $k \geq 0$. This is exactly true for operating the SDM with DC input, and approximately true during normal operation of the SDM.

In many practical situations, the magnitude of the state variables of the SDM should not be larger than certain values. For the direct form realization, since all the state variables are the delay versions of the output of the loop filter, we denote the bounds on the state variables as V_{cc}. That is, $|x_i(k)| < V_{cc}$ for $i = 1,2,\cdots,N$ and $k \geq 0$. Otherwise, the SDM is guaranteed to yield an unwanted behavior. Denote B_o as the set of allowable state vectors. That is, $B_o = \{\mathbf{x} : |x_i| < V_{cc} \text{ for } i = 1,2,\cdots,N\}$.

3. FUZZY IMPULSIVE CONTROL

Figure 2 shows the block diagram of how the proposed fuzzy impulsive controller connected to the SDM. We show in the following how we define the fuzzy membership functions.

First of all, for audio applications [3], if the norm of the difference between the original state vectors $\mathbf{x}(k_0)$ and the controlled state vectors $\mathbf{x}^c(k_0+1)$ is too large, audible clicks may appear. In order to minimize this effect, a fuzzy membership function is formulated to minimize the difference between the projection of the original state vectors and the controlled state vectors. Secondly, since limit cycle behavior should be avoided, another fuzzy membership function is formulated to suppress the occurrence of limit cycles. Finally, a fuzzy membership function is formulated to guarantee the local stability of SDMs.

Then a fuzzy impulsive control law can be formulated based on these membership functions. As a result, the controlled trajectory is guaranteed to be bounded for all initial conditions in the state space, no matter what the input step size is and the filter parameters are.

4. SIMULATION RESULTS

Consider a fifth order SDM with loop filter transfer function

$$\frac{20z^{-1}-74z^{-2}+103.0497z^{-3}-64.0015z^{-4}+14.9584z^{-5}}{1-5z^{-1}+10.0025z^{-2}-10.0075z^{-3}+5.0075z^{-4}-1.0025z^{-5}}. \quad (9)$$

This SDM can be implemented via the Jordan form [3] and can be realized as the following state space equation

$$\tilde{\mathbf{x}}(k+1)=\tilde{\mathbf{A}}\tilde{\mathbf{x}}(k)+\tilde{\mathbf{B}}(u(k)-y(k)) \quad (10)$$

for $k \geq 0$, where

$$y(k)=Q(\tilde{\mathbf{C}}\tilde{\mathbf{x}}(k)), \quad (11)$$

$$\tilde{\mathbf{A}} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -0.0018 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -0.000685 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \quad (12)$$

$$\tilde{\mathbf{B}} \equiv [1, \quad 0, \quad 0, \quad 0, \quad 0]^T, \quad (13)$$

and

$$\tilde{\mathbf{C}} \equiv [20, \quad 6, \quad 1, \quad 0.09375, \quad 0.00589]. \quad (14)$$

Assume that the initial condition is zero, that is $\tilde{\mathbf{x}}(0)=[0, \quad 0, \quad 0, \quad 0, \quad 0]^T$, we can check that the trajectory is bounded if the input step size u is approximately between -0.71 and 0.75, and may diverge if u is outside this range. By using a simple transformation, this SDM can be realized by the direct form and the corresponding initial condition is $\mathbf{x}(0)=[0, \quad -5, \quad 28.5, \quad 32.25, \quad 35.9793]^T$ when $u=0.75$. The relationship between the maximum absolute value of the state variables $\mathbf{x}$ and the input step size u is plotted in Figure 3. From the simulation result, we can see that even though the trajectory is bounded for a certain range of u, the maximum absolute value of $\mathbf{x}$ is between 20.0523 and 59.4633, which may be too large for some practical applications. If our proposed fuzzy impulsive control is applied at $V_{cc}=20$, then the maximum bounds on the state variables is guaranteed to be less or equal to 20, as shown in Figure 3, no matter what the input step size is.

It is worth noting that this SDM is *not* globally stable, that means, $\exists \tilde{\mathbf{x}}(0) \in \Re^N$ such that the trajectory is unbounded, for example, when $\bar{u}=0.75$, Figure 4a and Figure 4b show the responses of $x_1(k)$ for two initial conditions with a very small difference. It can be seen that even though the SDM exhibits acceptable behavior when $\tilde{\mathbf{x}}(0)=[0, \quad 0, \quad 0, \quad 0, \quad 0]^T$, the SDM can exhibit divergent behavior when $\tilde{\mathbf{x}}(0)=[0.001, \quad 0, \quad 0, \quad 0, \quad 0]^T$. If our proposed fuzzy impulsive control strategy is applied, the maximum absolute value of the state variables is always bounded by V_{cc} for $k>0$ and $\forall \mathbf{x}(0) \in \Re^N$, as shown in Figure 4c and 4d when $V_{cc}=40$.

To verify the independence of the filter parameters on the proposed fuzzy impulsive control strategy, consider another fifth order SDM with the following transfer function [3]

$$\frac{0.7919z^{-1}-2.8630z^{-2}+3.9094z^{-3}-2.3873z^{-4}+0.5498z^{-5}}{1-5z^{-1}+10.0023z^{-2}-10.0069z^{-3}+5.0069z^{-4}-1.0023z^{-5}}. \quad (15)$$

The trajectory of this SDM with $\bar{u} = 0.59$ and $\tilde{\mathbf{x}}(0) = [0, \ 0, \ 0, \ 0, \ 0]^T$ is shown in Figure 5a. It can be seen that the trajectory oscillates and diverges to infinity. On the other hand, when our proposed fuzzy impulsive control is applied, the maximum absolute value of the state variables is always bounded by V_{cc} for $k > 0$, $\forall a_i \in \Re$ for $i = 0,1,\cdots,N$ and $\forall b_j \in \Re$ for $j = 1,\cdots,N$. Figure 5b shows the corresponding state response when V_{cc} is set at 15.

5. CONCLUSIONS

In this paper, we have proposed a fuzzy impulsive control strategy for the stabilization of high order interpolative SDMs for audio applications. The main advantage of this control strategy is that the effective poles of the loop filter are not affected, and so the SNR performance of the SDMs is maintained or improved after the control. The controlled trajectory is guaranteed to be bounded no matter what the input step size is and what the filter parameters and the initial conditions are. The bounded region can also be altered easily. Examples of high order interpolative SDMs were given to demonstrate the effective performance of our proposed control strategy.

6. ACKNOWLEDGEMENTS

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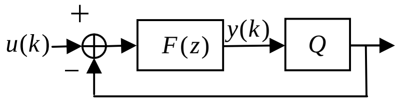

Figure 1. The block diagram of an interpolative SDM.

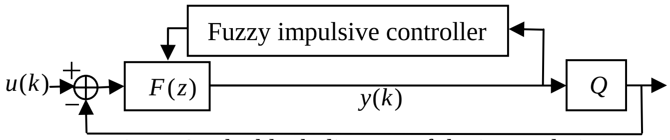

Figure 2. The block diagram of the interpolative SDM under the fuzzy impulsive control.

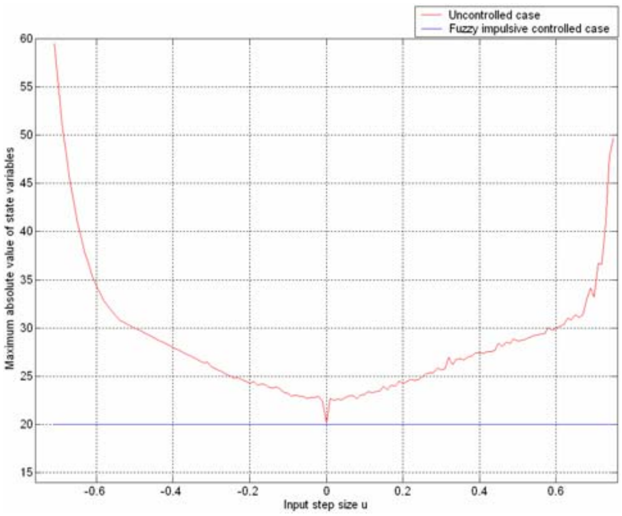

Figure 3: The relationship between the maximum absolute value of the state variables and the input step size when $\tilde{\mathbf{x}}(0) = [0, \ 0, \ 0, \ 0, \ 0]^T$.

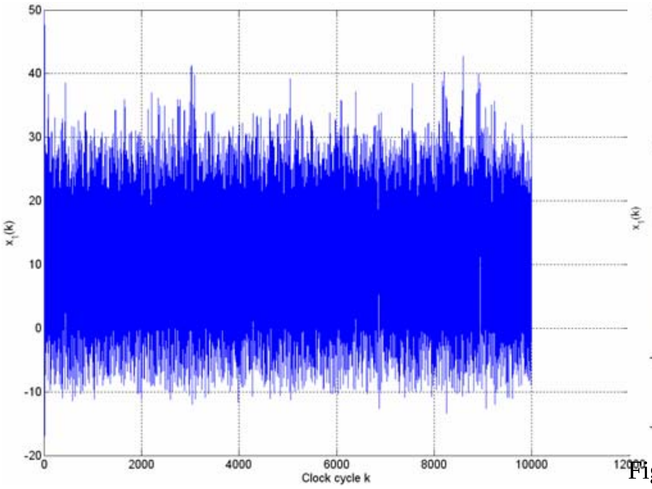

Figure 4a: The response of $x_1(k)$ with initial condition $\tilde{\mathbf{x}}(0) = [0, \quad 0, \quad 0, \quad 0, \quad 0]^T$ and input step size $\bar{u} = 0.75$ without our proposed fuzzy impulsive control strategy.

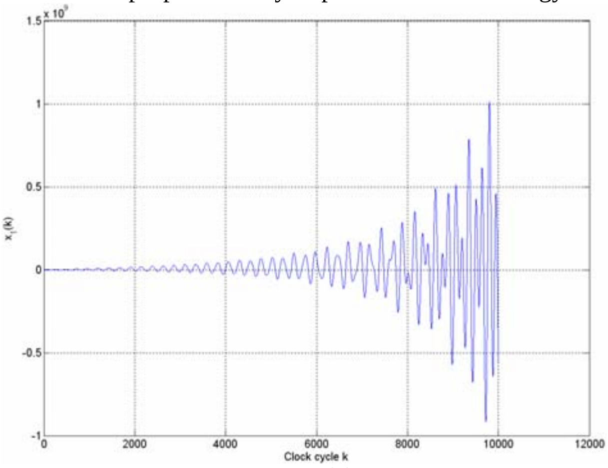

Figure 4b: The response of $x_1(k)$ with initial condition $\tilde{\mathbf{x}}(0) = [0.001, \quad 0, \quad 0, \quad 0, \quad 0]^T$ and input step size $\bar{u} = 0.75$ without our proposed fuzzy impulsive control strategy.

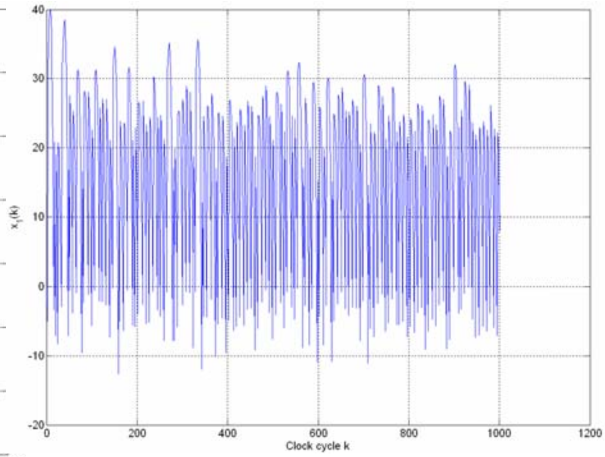

Figure 4c: The response of $x_1(k)$ with initial condition $\tilde{\mathbf{x}}(0) = [0, \quad 0, \quad 0, \quad 0, \quad 0]^T$ and input step size $\bar{u} = 0.75$ with our proposed fuzzy impulsive control strategy.

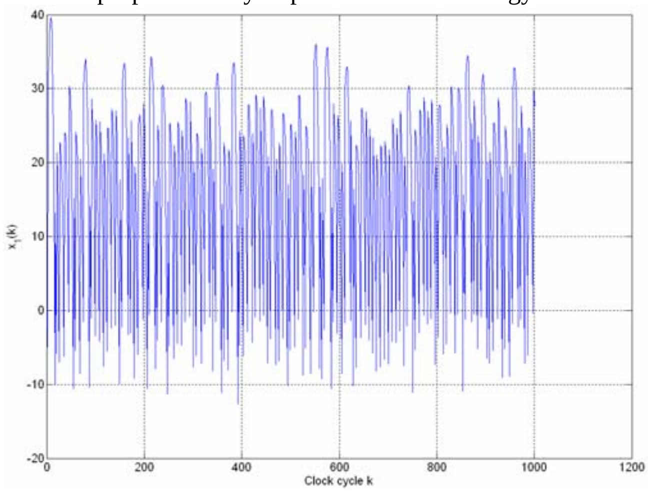

Figure 4d: The response of $x_1(k)$ with initial condition $\tilde{\mathbf{x}}(0) = [0.001, \quad 0, \quad 0, \quad 0, \quad 0]^T$ and input step size $\bar{u} = 0.75$ with our proposed fuzzy impulsive control strategy.

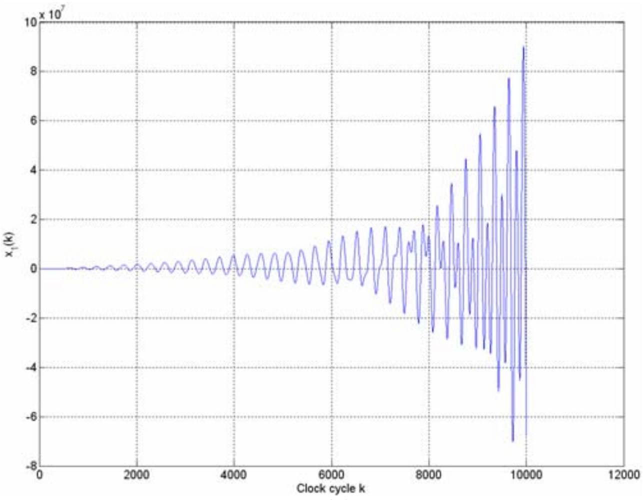

Figure 5a: The response of $x_1(k)$ with initial condition $\tilde{\mathbf{x}}(0) = [0, \quad 0, \quad 0, \quad 0, \quad 0]^T$ and input step size $\bar{u} = 0.59$ without our proposed fuzzy impulsive control strategy.

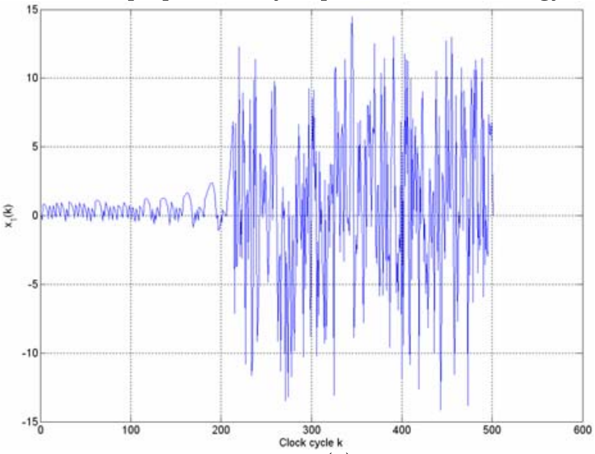

Figure 5b: The response of $x_1(k)$ with initial condition $\tilde{\mathbf{x}}(0) = [0, \quad 0, \quad 0, \quad 0, \quad 0]^T$ and input step size $\bar{u} = 0.59$ with our proposed fuzzy impulsive control strategy.