Many-to-Many Matching With Externalities for Device-to-Device Communications

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***Abstract*—In this letter, we aim to solve the resource allocation problem for device-to-device (D2D) communications underlaying cellular networks. Particularly, multiple D2D pairs are allowed to reuse the same resource block (RB), and one D2D pair is allowed to use the spectrum of multiple RBs. Our objective is to maximize the system sum rate by satisfying the signal-to-interference-plus-noise ratio constraints for both D2D and cellular user equipments. In order to solve this non-deterministic polynomial-time hard optimization problem, we propose a novel algorithm for obtaining a sub-optimal solution based on the many-to-many two-sided matching game with externalities. To characterize the properties of the proposed algorithm, we prove that it converges to the two-sided exchange stability within a limited number of iterations. Additionally, simulation results show that the proposed algorithm can achieve the near-optimal system sum rate and significantly outperforms a one-to-one matching algorithm.**

***Index Terms*—Device-to-device communications, externalities, many-to-many matching, resource allocation.**

I. Introduction

DEVICE-TO-DEVICE (D2D) communications is considered as one of the pieces of the fifth generation (5G) jigsaw puzzle in order to improve spectrum efficiency [1]. Driven by the potential benefits of D2D communications, many works have been prompted in different scenarios [2]–[4]. Solution approaches that allowed cellular devices and D2D pairs to share spectrum resources were proposed in [2], thereby increasing the spectrum efficiency of traditional cellular networks. In [3], D2D spectrum sharing and mode selection using a hybrid network model and a unified analytical approach were jointly studied. In [4], from the perspective of security issue, the performance of secure D2D communication was investigated in energy harvesting large-scale cognitive radio networks.

Although D2D promises unprecedented increase in spectrum efficiency, it brings in interference to the cellular network [5]. To tackle this issue, some researches on resource allocation have been motivated recently, where game theory is considered as one of the promising approaches [6], [7]. In [6], a reverse iterative combinatorial auction was introduced as the resource allocation mechanism, with the objective to maximize the system sum rate. A dynamic Stackelberg game framework was proposed in [7], jointly considering the mode selection and spectrum partitioning for D2D communications. Despite the potentials of game theory, it also has some shortcomings [8], including the distributed-limited implementation and unilateral equilibrium deviations, which makes it unpractical when solving some assignment problems. As such, matching theory [9]–[12] has been in the spotlight for wireless resource allocation, which can overcome some of the limitations of the game theory. The body of work in [9] and [10] was based on the classical deferred acceptance algorithm, where externalities among players were not taken into consideration. In [11], a one-to-one matching model with externalities was discussed. Pantisano *et al.* [12] formulated a many-to-one matching problem with externalities. However, the complexity for analyzing the stability of both the one-to-one and many-to-one matching game with externalities is much lower than that of the many-to-many one.

Different from the prior work, we propose a novel resource allocation approach based on the many-to-many matching game with externalities. By doing so, the resource utilization can be improved and the mutual interference among D2D pairs matched to the same RB can be well handled. The main contributions of this letter are summarized as follows. 1) We formulate the system sum rate maximization problem taking account of the signal-to-interference-plus-noise ratio (SINR) constraints for both D2D and cellular user equipments (UEs); 2) We model the formulated problem as a many-to-many matching game with externalities, and propose a novel algorithm of resource allocation for D2D communications to obtain a stable matching between the D2D pairs and RBs; 3) We prove that the proposed algorithm converges to a stable state within limited number of iterations; and 4) Simulation results show that RADMT can achieve the near-optimal performance compared to the exhaustive search, which significantly outperforms a one-to-one matching algorithm.

II. System Model and Problem Formulation

We consider a scenario of sharing uplink resources of the cellular network. Both the evolved NodeB (eNB) and UEs are equipped with a single omni-directional antenna. The eNB maintains the radio resource control for both cellular and D2D communications. The cellular UEs and D2D transmitters are distributed uniformly in the cell, while each D2D receiver obeys a uniform distribution inside the circle centered at the corresponding D2D transmitter, with a radius d_{max}. The set of D2D pairs is denoted by $\mathcal{D} = \{D_1, \ldots, D_i, \ldots, D_I\}$, and the set of D2D transmitters and receivers are denoted by $\{DT_1, \ldots, DT_i, \ldots, DT_I\}$ and $\{DR_1, \ldots, DR_i, \ldots, DR_I\}$, respectively. $\mathcal{RB} = \{RB_1, \ldots, RB_j, \ldots, RB_J\}$ is the set of RBs. For the sake of simplicity, we use the same index for cellular UEs with RBs, i.e., the set of cellular UEs is denoted by $\mathcal{C} = \{C_1, \ldots, C_j, \ldots, C_J\}$. The channel is modeled as

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Rayleigh fading where the channel response follows the independent complex Gaussian distribution.[1] Hence, the channel gain can be expressed as $G = \beta L^{-\eta}|h|^2$, where β is the system constant, L is the distance between signal transmitter and receiver, η is the path-loss exponent, and h is the complex Gaussian channel coefficient that obeys $h \sim \mathcal{CN}(0, 1)$.

We allow multiple D2D pairs to share the same RB and one D2D pair to occupy multiple RBs. We use the element α_{ij} to indicate whether a RB is allocated to a D2D pair or not. More specifically, if RB_j is allocated to D_i, $\alpha_{ij} = 1$; otherwise, $\alpha_{ij} = 0$. We assume that the total transmit power of each D2D transmitter is a fixed value and the power is equally divided over the occupying RBs. The power allocated to the D2D pair D_i over RB RB_j is denoted by p_i^j, satisfying $p_i^j = \frac{P_i}{\sum_{j=1}^{J} \alpha_{ij}}$, where P_i is the total transmit power of DT_i. Suppose that RB_j is allocated to D_i, then the received signal-to-noise-plus-interference-ratio (SINR) at DR_i on RB_j is $\gamma_i^j = \frac{p_i^j G_i}{Q_j G_{ji} + \sum_{i' \neq i} \alpha_{i'j} p_{i'}^j G_{i'i} + N_0}$, where Q_j is the transmit power of C_j. G_i, G_{ji}, $G_{i'i}$ are the channel gains between DT_i and DR_i, that between C_j and DR_i, and that between $DT_{i'}$ and DR_i, respectively. N_0 is the additive white Gaussian noise power. Similarly, the received SINR at the eNB is $\gamma_j = \frac{Q_j G_{jB}}{\sum_i \alpha_{ij} p_i^j G_{iB} + N_0}$, where G_{jB} and G_{iB} are the channel gain between C_j and the eNB, and that between DT_i and the eNB, respectively. Based on the Shannon-Hartley theorem, we can obtain the data rates of D_i on RB_j and that of C_j as $R_i^j = \alpha_{ij} B \log_2(1 + \gamma_i^j)$ and $R_j = B \log_2(1 + \gamma_j)$, respectively, where B is the bandwidth of a RB.

Our objective is to maximize the system sum rate with SINR constraints for both D2D and cellular UEs, which can be expressed as

$$\max_{\alpha_{ij}} \sum_j \sum_i \left(R_i^j + R_j\right), \tag{1a}$$

$$s.t. \quad \alpha_{ij}\gamma_i^j \geq \alpha_{ij}\gamma_i^{min}, \quad \forall i, j, \tag{1b}$$

$$\gamma_j \geq \gamma_j^{min}, \quad \forall j, \tag{1c}$$

$$\alpha_{i,j} \in \{0, 1\}, \quad \forall i \in \{1, \ldots, I\}, \forall j, \tag{1d}$$

$$\sum\nolimits_i \alpha_{i,j} \leq q_{max}, \quad \forall j, \tag{1e}$$

where γ_i^{min} and γ_j^{min} are the minimum SINR targets for D_i and C_j, respectively. (1b) and (1c) restrict the SINR requirements of D2D and cellular UEs. (1d) shows that the value of $\alpha_{i,j}$ should be either 0 or 1. In (1e), it is shown that at most q_{max} D2D pairs can be allocated to each RB. This constraint is to restrict the interference on each RB, as well as reduce the implementation complexity.

Note that the formulated problem is a non-convex one due to the binary constraints as well as the existence of the interference term in the objective function [13]. Therefore, it may be too complex to solve this problem by utilizing the conventional centralized exhaustive method, especially in a dense network. However, since (1) contains only one binary variable, it can be modeled as a matching problem. Thus to optimally solve the optimization problem (1), we will develop a many-to-many matching algorithm in the next section.

[1] Considering correlated coefficients in adjacent RBs is beyond of the scope of this letter.

III. Resource Allocation for D2D Communications Using Matching Theory

Matching theory is a promising approach to perform resource management in wireless networks [14]. The main benefit of matching theory is the ability to define individual utilities for D2D pairs and RBs as well as the self-organizing solution to the resource allocation problem.

A. Many-to-Many Matching With Externalities

In this letter, we consider the many-to-many matching model between the D2D pairs and RBs, which is defined as the following:

Definition 1: In the many-to-many matching model, a *matching* μ is a function from the set $\mathcal{RB} \cup \mathcal{D}$ into the set of all subsets of $\mathcal{RB} \cup \mathcal{D}$ such that 1) $|\mu(D_i)| \leq J, \forall D_i \in \mathcal{D}$, and $\mu(D_i) = \emptyset$ if D_i is not matched to any RB; 2) $|\mu(RB_j)| \leq q_{max}$, $\forall RB_j \in \mathcal{RB}$, and $\mu(RB_j) = \emptyset$ if RB_j is not matched to any D2D pair; 3) $RB_j \in \mu(D_i)$ iff $D_i \in \mu(RB_j)$.

We define the preference value for the D2D pair D_i on RB_j as $U_i(j) = R_i^j$. It is easy to find that $U_i(j)$ is a function of the interference from the D2D and cellular UEs occupying the same RB. Therefore, we can make the following observation:

Remark 1: The proposed matching game has *externalities*, where the preference values of D2D pairs not only depend on the RBs that they are matched with, but also on the other D2D pairs matched to the same RB.

This type of matching is called the matching game with externalities, where each player has a dynamic preference list over the opposite set of players. This is different from the conventional matching games in which players have fixed preference lists [9], [10], [14]. In this matching model, the preference of players over the opposite set of players replies on the matching states. Therefore, a preference list over the set of matching states is adopted. For example, the preference list of the D2D pair D_i on all the possible matching states is with respect to the descending order for the value of $U_i(j, \mu)$, where $U_i(j, \mu)$ is the utility of the D2D pair D_i on the RB RB_j under the matching state μ.

We define the preference value of RB_j on the set of D2D pairs $\mathcal{S}$ under the matching state μ as the sum rate of both the occupying D2D pairs as well as the corresponding cellular UE, i.e., $U_j(\mathcal{S}, \mu) = R_j + \sum_{i \in \mathcal{S}} R_i^j$. As with the preference lists of the D2D pairs, the preference list of a RB RB_j is ranked by RB_j's preference values in descending order.

Motivated by the housing assignment problem in [15], we propose an extended matching algorithm for the many-to-many matching problem with externalities. Different from the traditional deferred acceptance algorithm solution [14], the *swap operations* between any two D2D pairs to exchange their matched RBs is enabled. To better describe the interdependencies between the players' preferences, we first define the concept of *swap matching* as follows:

$$\mu_{ij}^{i'j'} = \left\{\mu \setminus \left\{(i, \mu(i)), \left(i', \mu(i')\right)\right\}\right\} \cup \left\{\left(i, \left\{\{\mu(i) \setminus \{j\}\} \cup \{j'\}\right\}\right), \left(i', \left\{\left\{\mu(i') \setminus \{j'\}\right\} \cup \{j\}\right\}\right)\right\}, \tag{2}$$

where $j \in \mu(i)$, $j' \in \mu(i')$, $j \notin \mu(i')$, and $j' \notin \mu(i)$. In other words, a swap matching enables D2D pair D_i and $D_{i'}$ to switch one of their matched RBs while keeping other D2D pairs and RBs' matchings unchanged. It is worth noticing that one of the D2D pairs involved in the swap can be a "hole" representing an open spot of a RB, thus allowing for a single D2D pair

TABLE I
RESOURCE ALLOCATION FOR D2D COMMUNICATIONS USING MATCHING THEORY (RADMT)

Step 1: Initialization

1) D2D pairs and RBs are randomly matched with each other subject to constraints (1b) - (1e).
2) Each D2D pair equally divides its transmit power on the matched RBs.

Step 2: Swap-matching process

1) For each D2D pair D_i, it searches for another D2D pair $D_{i'}$ or an open spot $\mathcal{O}$ of RB's available vacancies to form a swap-blocking pair.
   a) If $(D_i, D_{i'})$ or $(D_i, \mathcal{O})$ forms a swap-blocking pair along with $j \in \mu(i)$, and $j' \in \mu(i')$,
      i) update the current matching state to $\mu_{ij}^{i'j'}$.
      ii) update the number of D2D pairs matched with each RB.
   b) Else if there does not exist such a swap-blocking pair,
      i) keep the current matching state.
2) Repeat ***Step 2*** until there is no swap-blocking pair in the current matching.

Step 3: End of the algorithm.

moving to available vacancies. Similarly, one of the RBs RB_j involved in the swap can be a "hole" if $\mu(i) = \emptyset$.

Based on the concept of *swap matching*, the *swap-blocking* pair is defined as

Definition 2: $(D_i, D_{i'})$ is a *swap-blocking* pair if and only if 1) $\forall x \in \{i, i', j, j'\}$, $U_x(\mu_{ij}^{i'j'}) \geq U_x(\mu)$, and 2) $\exists x \in \{i, i', j, j'\}$, such that $U_x(\mu_{ij}^{i'j'}) > U_x(\mu)$.

The swap operations are expected to take place between the *swap-blocking* pairs. That is, if two D2D pairs want to switch between two RBs, the RBs involved must "approve" the swap. The condition (1) implies that the utilities of all the involved players should not be reduced after the swap operation between the *swap-blocking* pair $(D_i, D_{i'})$. The condition (2) indicates that at least one of the players' utilities is increased after the swap operation between the *swap-blocking* pair. This avoids looping between equivalent matchings where the utilities of all involved agents are indifferent. Note that the utilities of the "holes" and the players in the opposite set matched with the "holes" are not considered in these two conditions. Through multiple swap operations, the dynamic preferences of players which depend on the entire matching of the others, and the externalities of matchings are well handled.

As stated in [14], there is no longer a guarantee that a traditional "pairwise-stability" exists when players care about more than their own matching, and, if a stable matching does exist, it can be computationally difficult to find. Bodine-Baron *et al.* [15] focused on the *two-sided exchange-stable matchings*, which is defined as follows:

Definition 3: A matching μ is *two-sided exchange-stable* if there does not exist a *swap-blocking* pair.

The *two-sided exchange stability* is a distinct notion of stability compared to the traditional notion of stability of [14], but one that is relevant to our situation where agents can compare notes with each other.

B. Proposed Resource Allocation Algorithm

The proposed matching algorithm, i.e., RADMT, is shown in Table I. The algorithm consists of three main steps: Step 1 sets up the initial matching state; Step 2 focuses on the swap-matching process between different D2D pairs; and Step 3 outputs the final matching state. Initially, D2D pairs and RBs randomly match with each other satisfying constraints (1b) - (1e), and each D2D pair performs equal power allocation on its matched RBs. Subsequently, each D2D pair keeps searching for all the other D2D pairs and the available vacancies of RBs to check whether there is a swap-blocking pair. The swap-matching process ends when there exists no swap-blocking pair, and the final matching is obtained.

To evaluate the proposed algorithm, we analyze the properties in terms of effectiveness, stability, convergence, complexity and overhead in the following.

1) Effectiveness: The system sum rate increases after each swap operation.

Proof: Suppose a swap operation makes the matching state change from μ to $\mu_{ij}^{i'j'}$. According to RADMT, a swap operation occurs only when $U_j(\mu_{ij}^{i'j'}) \geq U_j(\mu)$ as well as $U_{j'}(\mu_{ij}^{i'j'}) \geq U_{j'}(\mu)$. Given that $U_j(\mu(j), \mu) = R_j(\mu(j), \mu) + \sum_{i \in \mu(j)} R_i(j, \mu)$, we have

$$\Phi_{\mu \to \mu_{ij}^{i'j'}} = R_{sum}\left(\mu_{ij}^{i'j'}\right) - R_{sum}(\mu)$$
$$= \sum_j \left(R_j\left(\mu_i^{i'j'}(ij), \mu_{ij}^{i'j'}\right) + \sum_{i \in \mu_{ij}^{i'j'}(j)} R_i\left(j, \mu_{ij}^{i'j'}\right)\right)$$
$$- \sum_j \left(R_j(\mu(j), \mu) + \sum_{i \in \mu(j)} R_i(j, \mu)\right) > 0, \quad (3)$$

where $\Phi_{\mu \to \mu_i^{i'}}$ is the difference of the system sum rates under the matching state $\mu_{ij}^{i'j'}$ and that under the matching state μ. ■

2) Stability: If the proposed algorithm converges to a matching μ^*, then μ^* is a two-sided exchange-stable matching.

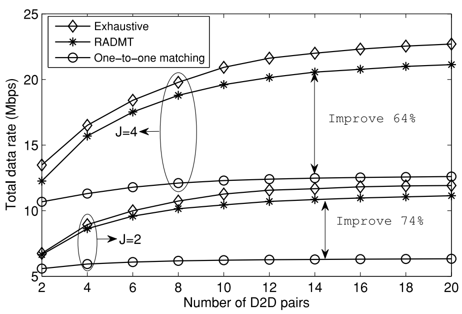
Proof: Assume that there exists a swap-blocking pair $(D_i, D_{i'})$ in the final matching μ^* satisfying that $\forall x \in \{i, i', j, j'\}$, $U_x(\mu_{ij}^{i'j'}) \geq U_x(\mu^*)$ and $\exists x \in \{i, i', j, j'\}$, such that $U_x(\mu_{ij}^{i'j'}) > U_x(\mu^*)$. According to Table I, the algorithm does not terminate until all the swap-blocking pairs are eliminated. To this end, μ^* is not the final matching, which causes conflict. Therefore, we conclude the proposed algorithm can reach the two-sided exchange stability in the end of the algorithm. ■

3) Convergence: From (3), we find that the system sum rate increases after each successful swap operation. Since the system sum rate has an upper bound due to limited spectrum resources, the swap operations stop when the system sum rate is saturated. Therefore, within limited number of rounds, the matching process converges to the final state which is stable.

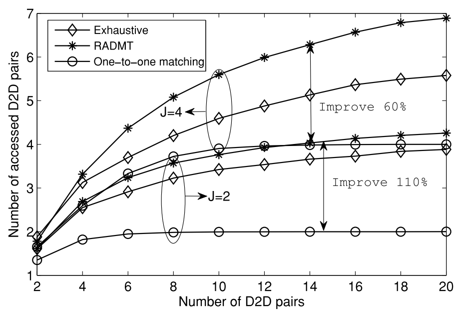
4) Complexity and Overhead: The number of communication packets between the D2D pairs and the RBs required in RADMT is upper bounded by $\mathcal{N}_{max} = \binom{I}{2} + I \times J$.

Proof: Following the RADMT in Table I, the D2D pairs and RBs communicate with each other in the swap-matching process to find the potential *swap-blocking* pairs. The number of communication packets of the potential swap operations between any two D2D pairs is $\binom{I}{2}$. Furthermore, the D2D pairs also search for the open spots of RBs' available vacancies to form *swap-blocking* pairs, and the maximum number of communication packets for this process is $I \times J$.

Regarding the time scale of the proposed algorithm, the signaling packet length required for the communication between the D2D pairs and the RBs until the algorithm converges is very short. In particular, each D2D pair is only required to send one bit to another D2D pair indicating a swap-operation offer, and then the involved D2D pairs each send a one-bit request to their occupying RBs. Finally, the RBs only need to



(a) System sum rate versus different number of D2D pairs.



(b) Number of accessed D2D pairs versus different number of D2D pairs.

Fig. 1. Performance analysis of the proposed algorithm.

send one bit back to the offering D2D pairs indicating either accept or reject the request. The total amount of overhead from the proposed algorithm thus can be quite small.

It can be observed that the complexity of the exhaustive searching method increases exponentially with the number of D2D pairs and RBs. In contrast, the complexity of the proposed algorithm is $O(I * J)$, which is significantly lower than that of the exhaustive searching method. ■

IV. Simulation Results and Discussion

In this section, numerical results are provided to demonstrate the performance of the proposed algorithm. The exhaustive optimal search and one-to-one matching algorithm are also plotted as benchmarks. Specifically, the exhaustive search guarantees the global optimal result and the one-to-one matching algorithm enables the one-to-one allocation of RBs to D2D pairs. For the simulations, we set the cellular radius to 300 m, the bandwidth of each RB to 180 kHz, the cellular UEs' SINR threshold to 4 dB, σ^2 to -98 dBm, the D2D UEs' SINR constraint to 2 dB, the total transmit power of D2D transmitters to 24 dBm, and d_{max} to 50 m.

Fig. 1(a) plots the system sum rate versus different numbers of D2D pairs. One can observe that the sum rate increases with the number of D2D pairs. When the number of D2D pairs is large enough, the sum rate keeps increasing due to the multi-user diversity gain, but with a lower speed. It is also observed that the proposed algorithm improves the sum rate by around 74% compared to the one-to-one matching algorithm in the case of $J = 2$, and 64% in the case of $J = 4$. Meanwhile, the proposed algorithm can reach 91.3% of the exhaustive optimal result, unequivocally substantiating the plausibility of the proposed algorithm.

Fig. 1(b) plots the number of accessed D2D pairs versus different numbers of D2D pairs in the network. With the increase of number of D2D pairs, the largest number of accessed D2D pairs is J in the one-to-one matching algorithm. This is because each RB can be allocated to no more than one D2D pair. The number of accessed D2D pairs of the proposed algorithm is improved by around 110% compared to that of the one-to-one matching algorithm in the case of $J = 2$, and 60% in the case of $J = 4$.

V. Conclusion

In this letter, a novel resource allocation algorithm was proposed for device-to-device (D2D) communications using many-to-many matching with externalities. It was demonstrated that the proposed algorithm could converge to a two-sided exchange-stable matching within limited number of iterations. Simulation results showed that the proposed algorithm achieved the near-optimal sum rate which significantly outperformed the one-to-one matching algorithm. This letter mainly focused on assigning cellular resources to D2D communications; our future work may consider the resource allocation for both cellular and D2D UEs as well as the power control for D2D UEs over multiple RBs.

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