ADI-FDTD algorithm in curvilinear co-ordinates

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The alternating direction implicit (ADI) scheme has been successfully applied to the finite-difference time-domain (FDTD) method to achieve an unconditionally stable algorithm. The ADI-FDTD method is extended to the curvilinear co-ordinate system to form an alternating direction implicit nonorthogonal FDTD (ADI-NFDTD) method. The numerical results show that the proposed ADI-NFDTD algorithm demonstrates better late time stability compared to the conventional NFDTD scheme.

Introduction: The finite-difference time-domain (FDTD) method [1] is a popular numerical technique in solving many electromagnetic problems. It can be generalised in the curvilinear co-ordinate system as a nonorthogonal FDTD (NFDTD) method [2, 3], which permits the analysis of arbitrary curved structures with fewer cells in conformal meshes. However, it has been proven that the NFDTD method suffers from late-time numerical instabilities and various efforts have been made to achieve longer stable simulation results [4, 5].

The alternating direction implicit (ADI) scheme has been successfully applied to the conventional Yee's FDTD and proven to be unconditionally stable [6]. In this Letter, the ADI-FDTD method is extended towards a more stable NFDTD scheme in curvilinear co-ordinates. Numerical experiments show that the ADI-NFDTD is stable over a much longer period of simulation compared to the conventional NFDTD algorithm.

Formulation: Numerical formulae for the proposed ADI-NFDTD method for 2-D TE wave are presented. As it is in the ADI-FDTD, an implicit iteration is applied in deriving the ADI-NFDTD and the calculation of one discrete time step is performed using two procedures. The first procedure is based on (1)–(5) and is taken as an example to demonstrate this algorithm.

In the first procedure, when calculating y-directional electronic displacement (D^y), the partial derivative of magnetic field (H_z) is performed with an implicit difference approximation of its unknown pivotal values at the $(n+1/2)$th time step. The x-directional electronic displacement (D^x) is calculated using an explicit central difference approximation with the known H_z values at the nth time step. And in the calculation of H_z, the partial difference of E field is performed implicitly with the y-directional component (E_y) at time step $(n+1/2)$ and explicitly with the x-directional component (E_x) at time step n. In other words, in the first procedure, the y-directional E fields (D^y, E_y) are synchronised with H_z. The second procedure processes in a similar way, only the x-directional E fields (D^x, E_x) are synchronised with H_z. The second procedure is not presented here.

First procedure:

$$\frac{D^{x^{n+1/2}}(i+1/2,j) - D^{x^n}(i+1/2,j)}{dt/2} = \frac{H_z^n(i+1/2,j+1/2) - H_z^n(i+1/2,j-1/2)}{\sqrt{g(i+1/2,j)}} \quad (1)$$

$$\frac{D^{y^{n+1/2}}(i,j+1/2) - D^{y^n}(i,j+1/2)}{dt/2} = \frac{H_z^{n+1/2}(i+1/2,j+1/2) - H_z^{n+1/2}(i-1/2,j+1/2)}{\sqrt{g(i,j+1/2)}} \quad (2)$$

$$\frac{H_z^{n+1/2}(i+1/2,j+1/2) - H_z^n(i+1/2,j+1/2)}{dt/2} = \frac{(E_x^n(i+1/2,j+1) - E_x^n(i+1/2,j)) - (E_y^{n+1/2}(i+1,j+1/2) - E_y^{n+1/2}(i,j+1/2))}{\mu(i,j)\cdot\sqrt{g(i+1/2,j+1/2)}} \quad (3)$$

where μ is the medium permeability, g and the following g_{xx}, g_{yy}, g_{xy} in (4) and (5) are the computed metric tensor components from the local curvilinear co-ordinates. As is the case in the nonorthogonal FDTD scheme, the covariant field components (E_x and E_y) must be calculated from the contravariant displacement (D^x and D^y) by the interpolating equations:

$$Ex^n(i+1/2,j) = \frac{g_{xx}(i,j)\cdot D^{x^n}(i+1/2,j)}{\varepsilon_x(i,j)} + \frac{g_{xy}(i,j)}{4} \times \frac{(D^{y^n}(i,j-1/2) + D^{y^n}(i+1,j-1/2) + D^{y^n}(i,j+1/2) + D^{y^n}(i+1,j+1/2))}{\varepsilon_x(i,j)} \quad (4)$$

$$Ey^{n+1/2}(i,j+1/2) = \frac{g_{yy}(i,j)\cdot D^{y^{n+1/2}}(i,j+1/2)}{\varepsilon_y(i,j)} + \frac{g_{xy}(i,j)}{4} \times \frac{(D^{x^{n+1/2}}(i-1/2,j) + D^{x^{n+1/2}}(i+1/2,j) + D^{x^{n+1/2}}(i-1/2,j+1) + D^{x^{n+1/2}}(i+1/2,j+1))}{\varepsilon_y(i,j)} \quad (5)$$

where ε_x and ε_y are the permittivities for calculating E_x and E_y separately.

Note that, in this interpolating scheme, E field values are calculated by the electric displacement (D) values from the same time step. By substituting the expressions for E_x^n and $E_y^{n+1/2}$ into (3) and the resulting expression for $H_z^{n+1/2}$ into (2), one can obtain a tridiagonal system of equation that can be easily solved. The $H_z^{n+1/2}$ in the first procedure can be updated in a similar way.

Numerical results: To demonstrate the validity of the proposed ADI-NFDTD algorithm, a cylindrical perfect electric conductor (PEC) cavity resonator is modelled with both the conventional NFDTD and the ADI-NFDTD algorithm. The radius of the resonator is 0.15 m and the computational region is 0.6 × 0.6 m meshed by 30 × 30 cells. The temporal results of magnetic field (H_z) at one point inside the cavity using the conventional NFDTD and the ADI-NFDTD methods are compared in Fig. 1. It can be seen that the conventional NFDTD result becomes unstable after 12 000 time steps, while the ADI-NFDTD result is stable up to time step 100 000. This demonstrates that the ADI-NFDTD is more stable compared to the conventional NFDTD scheme.

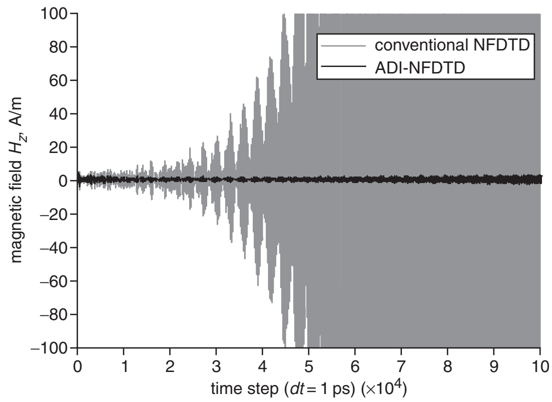

Fig. 1 *Comparison of H field time-domain results of one probe inside cavity*

More simulations are obtained with this model using different time intervals (dt). Frequency spectra are calculated by applying the fast fourier transform (FFT) to the temporal signatures. The resonant frequency spectra calculated from both the NFDTD and the ADI-NFDTD algorithm with dt equal to 1 ps are compared in Fig. 2. As has been mentioned before, for this dt value, after 12 000 time steps, the energy of the field increases gradually with the time. Under the consideration of both the stability and the FFT resolution, the total temporal samples used for FFT are the first 40 000 results, which mean that some parts of the unstable results are involved. With the ADI-NFDTD method, the number of temporal samples is chosen to be 100 000. So, the frequency resolution of the spectra for the conventional NFDTD and ADI-NFDTD are 0.025 and 0.01 GHz, respectively. It is observed that, with each dt, the frequency-domain results from the conventional NFDTD algorithm suffer from a coarse FFT resolution because of a short time signature. For both cases, Fig. 3 shows the relative error rates of numerical results (compared to the theoretical prediction) against the relative time interval (*dt*). It can be seen that, from the 16th unit, the relative error rate of the conventional NFDTD result increases dramatically. That is the *dt* for the CFL condition. From there on, any increase in *dt* results in the resonant frequencies becoming undetectable in the conventional NFDTD scheme; whereas in the ADI-NFDTD scheme, the error rate only increases slowly with increase in *dt*. It validates the efficiency of the ADI-NFDTD scheme.

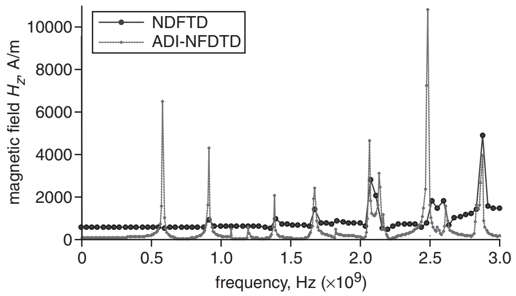

Fig. 2 *Resonant frequency spectra of cavity resonator calculated from two schemes with dt = 1 ps*

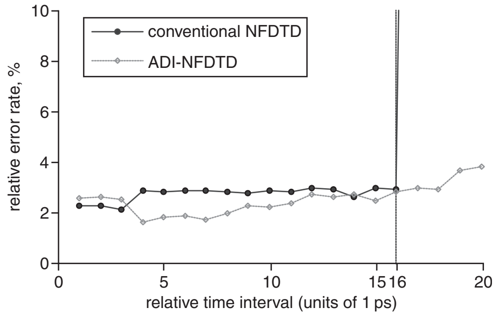

Fig. 3 *Relative error rate of resonant frequency spectra from conventional NFDTD and proposed ADI-NFDTD scheme against relative time interval dt/Δt (Δt = 1 ps)*

Conclusions: A novel scheme with the ADI method applied to the nonorthogonal FDTD algorithm is presented. The numerical results show that the CFL condition of the conventional NFDTD algorithm is removed by using the ADI-NFDTD scheme. The chosen *dt* value is only limited by the accuracy requirement. The late time stability of NFDTD is largely improved with this algorithm at the price of intensive calculation.

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