

Stable non-orthogonal FDTD method

V. Douvalis, Y. Hao and C.G. Parini

A novel approach for implementing a stable nonorthogonal finite-difference time-domain algorithm is proposed. Instead of representing a vector field with its contravariant and covariant components, this approach only makes use of the covariant components to annihilate any unstable behaviour. The new algorithm is validated by numerical experiments.

Introduction: A number of techniques have been proposed to extend the classical Yee's finite-difference time-domain (FDTD) algorithm to a grid that is conformal to curved objects. Such methods, including the non-orthogonal FDTD (NFDTD) [1, 2], can accurately represent the curved boundaries without resorting to inefficient staircase orthogonal meshes. However, their significant advantage is offset by the fact that they suffer from late time instabilities. In [3], it is demonstrated that the instability of NFDTD is inherent. Although a 'time-subgridding' approach is presented there to reduce the late time instability, an unconditionally stable NFDTD is still not obtained. In this Letter, we introduce a novel approach to discretise Maxwell's equations using only the covariant components. It is demonstrated that the new NFDTD algorithm is stable over a very long period of numerical simulation. Moreover, the algorithm is less computationally intensive than other NFDTD methods [1, 2] where both of the covariant and contravariant components are needed. The proposed technique is validated by simulation results on a circular waveguide resonator.

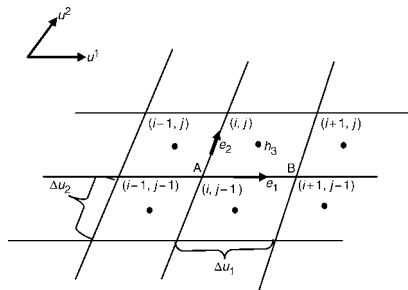


Fig. 1 Position of field components in new NFDTD scheme

Formulation: It has been proved that the source of late time instabilities in the NFDTD method is due to the basic definitions of the projection operators [4]. The contravariant-to-covariant field projection in the NFDTD algorithm is performed using a simple linear average. For example, the projection on the covariant electric component D_i is expressed as

$$D_i = \sum_{j=1}^3 g_{ij} \cdot f(D^j) \quad (1)$$

where g_{ij} are the computed metric tensor components from the local curvilinear co-ordinates and $f(D^j)$ is a function involving the contravariant fields D^j . In some cases f can be determined so as to produce stable results but the majority of such subsequent algorithms are based on unstructured meshes that are not suitable for NFDTD methods. At any rate, a scheme that can provide unconditional numerical stability for the NFDTD algorithm does not exist.

In this Letter, a new approach is proposed by the use of only covariant fields instead of using both contravariant and covariant components, which proves to be problematic in (1). The first step to achieve this is to discretise Maxwell's equations in the non-orthogonal co-ordinates. For a set of general co-ordinates (u^1, u^2, u^3) with unitary vectors $(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ they can be discretised as (for simplicity Ampere's law is written in the case of a 2D TE mode, g is the metric tensor as defined in [2]):

$$\epsilon \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{\sqrt{g}} \cdot \left[\frac{\partial h_3}{\partial u^2} \cdot \vec{a}_1 - \frac{\partial h_3}{\partial u^1} \cdot \vec{a}_2 \right] \quad (2)$$

In the non-orthogonal co-ordinates, three unitary vectors define its reciprocal basis vectors $(\vec{a}^1, \vec{a}^2, \vec{a}^3)$ and they satisfy the condition

$$\vec{a}^i \cdot \vec{a}_j = \delta_{ij} \quad (3)$$

where δ_{ij} is the commonly used symbol denoting unity when $i=j$ and zero when $i \neq j$. Also a vector can be presented either by its covariant or

contravariant components:

$$\vec{E} = \sum_i e_i \cdot \vec{a}^i = \sum_j e^j \cdot \vec{a}_j \quad (4)$$

There are two ways to expand (2) as a central difference equation for the purpose of implementing the NFDTD algorithm:

i) Take the dot product of (2) with the reciprocal basis vectors. This, due to (3) and (5), yields:

$$\epsilon \cdot \frac{\partial e^1}{\partial t} = \frac{1}{\sqrt{g}} \cdot \left[\frac{\partial h_3}{\partial u^2} \right] \quad (5)$$

The other equation concerning electric field components is obtained by straightforward index permutation. Equation (5) can be approximated in a central difference manner as usual and in its final form it will be the traditional non-orthogonal FDTD equation for the E-field, associating the contravariant e^1 -component with the covariant h_3 -component.

ii) However, an alternative is to take the dot product of (2) with the unitary basis vectors. This, due to (3), (4) and because $\vec{a}_i \cdot \vec{a}_j = g_{ij}$ [2], yields:

$$\epsilon \cdot \frac{\partial e_1}{\partial t} = \frac{1}{\sqrt{g}} \cdot \left[\frac{\partial h_3}{\partial u^2} \cdot g_{11} - \frac{\partial h_3}{\partial u^1} \cdot g_{12} \right] \quad (6)$$

As usual, (6) can be approximated as a central difference equation on the non-orthogonal grid. To this end, Fig. 1 shows the placement of the various components. They occupy the same places as the standard non-orthogonal FDTD formulation but this time the contravariant components are missing. Owing to the central difference scheme the derivative $\partial/\partial u^2$ along the u^2 -line will become

$$\frac{\partial h_3}{\partial u^2} = \frac{h_3(i, j) - h_3(i, j-1)}{\Delta u_2} \quad (7)$$

The $\partial/\partial u^1$ derivative is more complex to discretise since there are no h_3 -components along the u^1 -line. Therefore an averaging scheme must be employed. The averaging scheme is not expected to affect the stability of the algorithm because no contravariant components are included, but it could influence the accuracy of the method. This is going to be concluded by the simulation results. The chosen scheme must evaluate the h_3 component at the grid edges between which the electric field flows (for example points A and B) in Fig. 1. For points A and B it can be written as:

Point A: $h_3(A) = (1/4) \cdot (h_3(i, j) + h_3(i-1, j) + h_3(i, j-1) + h_3(i-1, j-1))$

Point B: $h_3(B) = (1/4) \cdot (h_3(i+1, j) + h_3(i, j) + h_3(i+1, j-1) + h_3(i, j-1))$

Owing to these relations the derivative becomes:

$$\frac{\partial h_3}{\partial u^1} = \frac{h_3(B) - h_3(A)}{\Delta u_1} = \frac{h_3(i+1, j) + h_3(i+1, j-1) + h_3(i-1, j) + h_3(i-1, j-1)}{4 \cdot \Delta u_1} \quad (8)$$

The formulation of other field components follows by analogy and a new scheme for the NFDTD algorithm is completed.

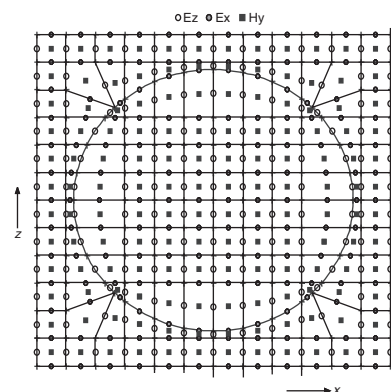


Fig. 2 Circular resonator meshed with non-orthogonal FDTD grid

Results: The new scheme is tested on a perfect electric conductor (PEC) circular resonator of Fig. 2. The resonator radius is 0.15 m and the computational region is 0.38×0.38 m meshed by 12×12 cells. The stability of the new algorithm is demonstrated in Fig. 3, where time samples of the magnetic field inside the resonator are shown. The algorithm remains stable even when the number of the time steps reaches 180 000. In contrast, when the conventional nonorthogonal FDTD method is applied using the same time step, the simulation becomes unstable after 10 000 steps. The proposed technique has also been tested for its accuracy. The simulated resonant frequencies are compared with the theoretical values as well as with the results yielded from a traditional NFDTD algorithm (Table 1). The overall agreement is very good.

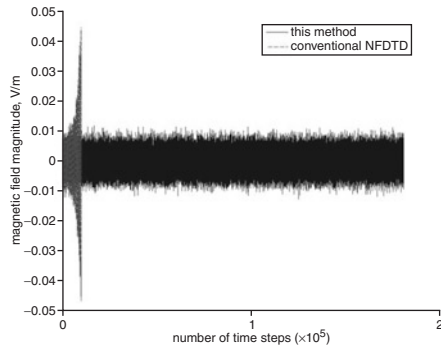


Fig. 3 Stable magnetic field signature in time domain with new method compared to traditional NFDTD scheme

Conclusions: It has been demonstrated that the proposed new algorithm for NFDTD calculations can annihilate the phenomenon of late time instabilities. Moreover, the use of one type of field components (covariants) leads to a new scheme that requires less computer resources in numerical simulations. However, its accuracy is not compromised as it is at the same high level as the traditional NFDTD schemes.

Table 1: Numerical results with novel and conventional NFDTD methods compared to theory

Theoretical results (GHz)	NFDTD scheme (GHz)	NFDTD error (%)	New scheme (GHz)	New scheme error (%)
0.5863	0.588	0.29	0.603	2.8
1.2198	1.18	-3.27	1.2	-1.65
1.7	–	–	1.7	0
2.043	2.03	-0.64	2.073	1.46
2.136	2.11	-1.22	2.183	2.2
2.553	2.53	-0.9	2.545	-0.3
2.718	2.65	-2.5	2.709	-0.3
2.732	2.76	1.02	2.726	-0.2

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